AN ASSESSMENT OF THE VALUE OF INFORMATION FROM ON-FARM FIELD TRIALS

David Bullock[1] and Taro Mieno[2]

Senior authorship not implied.

[1] Professor, Department of Agricultural and Consumer Economics, University of Illinois Urbana-Champaign
email: *dsbulloc@illinois.edu*.

[2] Assistant Professor, Department of Agricultural Economics, University of Nebraska Lincoln
email: *tmieno2@unl.edu*.

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Abstract

Recent research has used precision agricultural technology (PAT) to run large-scale field trials that greatly reduce the cost of data generation. We explore the economic value of such trials. We report results of Monte Carlo simulations in which information generated in trials is used to improve farm management in subsequent years. A key result is that in our simulations the value of PAT is not based on its making variable rate management feasible. Rather, most of the value of PAT comes from the improved management made possible by the information derived from the field trials the technology permits.

Precision agricultural technology (PAT)[1] has been commercially available for over fifteen years. The idea of using truly space-age methods to customize input management to small parts of farm fields was then and still is exciting. However, actual adoption of PAT remains far below projections of an enthusiastic farm and popular media of the 1990s.

Addressing this subject amidst the early excitement, Bullock et al. (1998), Bullock and Bullock (2000), and Bullock, Lowenberg-DeBoer, and Swinton (2002) predicted that PAT would be neither profitable nor widely adopted by commercial grain farmers in the then-foreseeable future. They argued that the complementarity between information about yield response and PAT made it infeasible to use the new technology without much more information than was available. That is, scarcity of data about the relationships between yields and factors of production has limited the demand for PAT. Bullock et al. (2009) provided empirical support for this hypothesis.

In recent experiments, researchers at the University of Illinois have shown that though the demand for PAT has been limited by a scarcity of data on yield response, that exact same technology can be used to run large-scale, on-farm field trials that supply the types of information needed to increase the profitability and demand for PAT. That is, PAT can be used to supply information needed to create more demand for PAT. We explain in the next section how such large-scale, on-farm field trials are currently being designed and run.

The objective of this article is to explore the economic value that can be created by generating data from such field trials. We report the results of Monte Carlo simulations in which such field trials are designed and run, and then the information generated is used to improve farm management in subsequent years. To the best of our knowledge, no previous economic studies have examined this economic problem, principally because the kind of large-scale field trials that we are modeling have only recently been conducted. In collaboration with others, the authors of this article are in the process of running hundreds of such trials over the next few years, and the simulations reported here offer a preview into the types of empirical results they may find. This preview provides significant insight into the possible future feasibility of making farm management significantly more reliant on data and statistical analysis, even at the site-specific level. With our simulations we examine: 1) whether the value of the information generated by such field trials is likely to pay for the costs of conducting the experiments; 2) whether having access to the types of information from our experiments can make PAT profitable; and 3) the optimal number of years to run whole-field experiments before beginning to use the data generated to improve management of the field.

Background: A New Way to Run On-Farm, Large-scale Field Trials

Agricultural scientists have been running agronomic field trials for over 160 years (O'dell, et al. 1982). The immediate purpose of field trials has always been to gather data on how input application management affects crop yields; the ultimate purpose has been to analyze that data to offer farmers improved recommendations about input management. Agricultural economists have had a rich history of using field trial data to estimate economically optimal management strategies. Early Heady famously conducted and analyzed dozens of such trials, primarily in the 1950s, 60s, and 70s (Heady and Pesek, 1954; Heady et al., 1955; Heady, 1957; Heady et al., 1964; Hexem, Sposito, and Heady, 1976). A lively debate about optimal nitrogen management and the functional forms of crop yield response functions has appeared in major agricultural economics journals over the past several decades (Swanson, Taylor, and Welch, 1973; Grimm, Paris, and Williams, 1987; Frank, Beattie, and Embleton, 1990; Paris, 1992; Bullock and Bullock, 1994; Chambers and Lichtenberg, 1996; Llewelyn and Featherstone, 1997; Tumusiime et al., 2011; Brorsen and Richter, 2012).

Despite this long history of scientific investigation into how crop yields respond to various factors of production, we will maintain in this article that to date relatively little is known about this subject. We contend that little is known because insufficient data has been generated to estimate yield response with sufficient statistical confidence in any kind of general way. For until recently, trials have had to be conducted using extremely labor-intensive techniques, for example by researchers marking off small plots of land using measuring tapes and flags, applying inputs by hand at varying rates on different plots, and harvesting without the benefit of large-scale farm machinery. This labor intensity meant that it was only financially feasible to run trials on very small areas of land, at few locations, and usually only for a few years. Furthermore, these experiments have almost entirely been run by different researchers with different experimental designs, making it difficult to combine the data for more reliable statistical analysis. Consequently, despite generations of research, university- and industry-provided fertilizer management recommendations have been based on "rules-of-thumb" (e.g., Hoeft and Peck, 2007) that are themselves, at best, based only loosely on data analysis and science (Rodriguez, 2014).

In the current article, we analyze a technological breakthrough in agronomic experimentation that we believe may soon significantly change how agronomic field trials are conducted, and will generate vast amounts of data from agronomic experiments, the analysis of which will significantly improve the management recommendations that science can provide farmers. Over the past several years, researchers at the University of Illinois, working both in the U.S. and with South American collaborators, have proved the concept of using PAT to run on-farm, large-scale field trials to efficiently and inexpensively gather very large amounts of data on how crop yields respond to input application rates, field characteristics, and weather (Casanoves, Macchiavelli, and Balzarini, 2007; Bullock et al., 2009; Peralta et al., 2013). Figure 1 illustrates a corn trial run in 2014 by University of Illinois crop scientists Donald Bullock and Robert Dunker on a 160-acre field. Using Enhanced

Farm Research Analysis (EFRA) software, which he co-designed, Don Bullock designed the agronomic field trial in less than 30 minutes, following rigorous spatial-statistical principles to examine the effects of nitrogen fertilizer application rates and seed rates on crop yield. Bullock used EFRA to design and then pre-program a variable application rate "map" into a computer aboard farm machinery. That program "instructed" application equipment to apply inputs at randomized rates on the 480 plots, each approximately one-third acre in size, while the participating farmer simply drove his equipment through the field in the usual manner. At harvest, a monitor was used to record yield on every plot, again with minimal bother to the farmer. Therefore, because the running of the experiment was almost entirely automatized, with minimal labor requirements, the new technique generated data at a fraction of the expense of gathering data from small-plot trials.

The types of field trials we examine in this article are not the only whole-field trials being used in research, nor the only ones implemented using precision technology. For example, researchers from seven Midwestern U.S. universities who are promoting their Maximum Return to Nitrogen approach have run many whole-field nitrogen corn trials over the past decade. But their trials have been strip trials, in which every unit of observation is a strip along the entire length of the field (one-half mile in a typical square 160-acre quarter section). In contrast, as we discuss in following sections, we implement "checkerboard" field trials to gather data from cells between about 200 and 300 feet long. Thus, in every experiment, our methodology gathers many times the observations obtainable through strip trials. In addition, the implicit statistical assumption in field trials is that field characteristics in a unit of observation are homogeneous. Since our units of observation are far smaller than whole field strips, in general the error brought about by the assumptions will be less of a problem for us than for those using the strip trial methodology.

Previous Studies and a Changing Approach to Field Trial Research

Since its commercial appearance in the 1990s, many studies have investigated the economic viability of PAT in farm management. As Bullock et al. (2009) briefly mention, frequently those studies have confounded the value of information needed to manage PAT with the value of the technology itself. For example, Koch et al. (2004) and Boyer et al. (2011) implemented agronomic experiments to examine the profitability of particular variable rate application (VRA) and uniform rate application (URA) fertilization strategies. However their conclusions about the strategies' relative profitabilities do not consider that both of the VRA and URA strategies used in their experiments are economically suboptimal, nor that their estimates of the values of each depend crucially upon how well their management strategies approximate optimal management strategies. Their estimates reflect less about the values of the technologies themselves than about how near their URA and VRA strategies happened to be to the economically optimal strategies.

Other economic studies compare the profitability of optimal VRA and URA strategies (Babcock and Pautsch, 1998; Thrikawala et al., 1999; Bullock, Lowenberg-DeBoer, and Swinton, 2002; Roberts et al., 2002; Wang et al., 2003; Lambert, Lowenberg-Deboer, and Malzer, 2006). Though the spatial scale and quality of data vary among studies, all first estimate/assume some yield response function of N. Then they solve the profit maximization problem for both VRA and URA strategies, with some studies more elaborate in their estimates of VRA costs than others.[2] Finally, they compare the maximized profits to conclude whether VRA is more profitable than URA. The key assumption underlying these studies is that farmers know their production function precisely, and thus they are already capable of implementing optimal URA and VRA strategies. The studies' conclusions about the economic viability of VRA are economically meaningful only if farmers have accurate knowledge of their farms' site-specific yield response functions.

In this article, we directly address the issue of the individual and joint values of PAT and information. In particular, we consider the value of PAT used in conjunction with yield response data gleaned from our large-scale, on-farm checkerboard field trials.

Conceptual Framework

Yield Response

Following (Bullock and Bullock, 2000), we assume the existence of a deterministic "meta yield response function", $f(\mathbf{x},\mathbf{c},\mathbf{z})$, which is dependent on three categories of variables: K managed inputs $\mathbf{x} = (x^1,\ldots,x^K)$[3], N spatially stochastic unmanaged factors of production (called "field characteristics," such as clay content, ground slope, etc.) $\mathbf{c} = (c^1,\ldots,c^N)$, and M temporally stochastic non-managed factors of production (called "weather") $\mathbf{z} = (z^1,\ldots,z^M)$. We also assume that output is subject to a random disturbance term, so that quantity yielded depends on the (deterministic) yield response function and the stochastic disturbance: $q = f(\mathbf{x},\mathbf{c},\mathbf{z}) + \varepsilon$.

The Field, Its "Maps" and Management Plan

To capture the essence of spatially-specific experimentation and management, we model a farm field with spatially heterogeneous field characteristics. Without significant loss of generality, we assume that the field is comprised of I rectangular, equally-sized plots. The field's characteristics are assumed to not vary within a plot, but to vary among plots. For plot $i = 1,\ldots,I$, we call the characteristics vector $\mathbf{c}_i = (c_i^1,\ldots,c_i^N)$ and call the management vector $\mathbf{x}_i = (x_i^1,\ldots,x_i^K)$. This field's characteristics map is $(\mathbf{c}_1,\ldots,\mathbf{c}_I)$ and its management map in year t is $(\mathbf{x}_{1t}, \ldots, \mathbf{x}_{It})$. Weather in year t is $\mathbf{z}_t = (z_t^1, \ldots, z_t^M)$. The field's yield disturbance map in year t is $\varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{I,t})$. Its yield map in year t is an I vector, with generic entry i being $f(\mathbf{x}_{it}, \mathbf{c}_{it}, \mathbf{z}_t) + \varepsilon_{it}$.

States of Nature

We let $\Omega_{\mathbf{c}}$ denote the set of all conceivable characteristics maps, Ω_{ε} denote the set of all conceivable yield disturbance maps, and $\Omega_{\mathbf{z}}$ denote the set of all conceivable levels of the vector of weather variables $\mathbf{z}$. We let Ω_f be the set of all conceivable response functions f, Ω_p be the set of all conceivable output price levels p, and let $\Omega_{\mathbf{w}}$ be the set of all conceivable input price vectors $\mathbf{w}$.[4] Then a generic state of nature is $\omega = (f, \mathbf{c}, \mathbf{z}, p, \mathbf{w}, \varepsilon)$, and the set of all states of nature is $\Omega = \Omega_{\mathbf{c}} \times \Omega_{\mathbf{z}} \times \Omega_f \times \Omega_p \times \Omega_{\mathbf{w}} \times \Omega_{\varepsilon}$. The objective joint probability density function of the states of nature is $g(\omega)$.

Research Projects and Information Structures with Noise

We assume the existence of a "farmer" (or "producer") who makes decisions about the levels of the managed input variables $\mathbf{x}$. In general, when the farmer chooses a production management plan, (s)he may not know with certainty the state of nature. There may be aspects of the state of nature that have been established and are known with certainty at decision-making time (for example, the price of fertilizer may be known when the decision to purchase fertilizer is made.) There may be aspects of the state of nature that have been established before decision-making time, but about which the the farmer lacks information. (For example, a farmer might not know perfectly the field's characteristics map, even though those characteristics will not change between decision-making time and the crop harvest.) Other aspects of the state of nature may be temporally stochastic, and not established until after the decision has been made. (For example, it is cost-prohibitive for farmers in the U.S. Corn Belt to fertilize their crop in August. But after all fertilization decisions have been made, rainfall in August affects yields.)

We consider that knowledge about probabilities that states of nature will occur can be improved by *research projects* that produce *information structures with noise*, as defined in Laffont (1989). In our simulations, research projects include agronomic field trials that generate data useful for the estimation of the meta-response function $f(\mathbf{x},\mathbf{c},\mathbf{z})$. Other research projects might include, for example, taking soil samples to gather information about the field's characteristics map, or developing meteorological models for more accurate estimates of probabilities of various weather events $\mathbf{z}$. We consider that a research project also involves analysis of the data gathered, including various econometric estimation procedures and tests. A generic set of "results" from a research project is a signal y (typically including tables of point estimates, hypothesis test results, etc.). Because some variables that affect crop yields are temporally stochastic, under different states of nature identical research procedures can produce different results, and so different signals. In addition, experiments may be imperfectly designed, and researchers might use sub-optimal econometric modeling techniques. Therefore generally a signal from a research project does not give the decision maker perfect knowledge of the state of nature. Formally, an information structure with noise is a conditional probability function, say $v(y|\omega)$, that reflects the probability with which a signal y out of the set of possible signals Y will be sent given a state of nature $\omega \in \Omega$ is drawn.

We assume that there is a baseline ("prior") information structure with noise provided by all past research. Calling all past research "Research Project 0" (or RP_0), the set of possible signals that might be sent, Y_0, contains only one element, which we will call y_0. The signal may be thought of as a message that says, "we can't tell you more than you know already." Beliefs in the baseline situation about the probabilities of states of nature being drawn are representative by a probability density function, $\pi(\omega)$, which for consistency with later notation we will also call $v_0(\omega|y_0)$. These prior beliefs are the decision maker's initial approximation of $g(\omega)$, the true probability density function of the states of nature. The farmer assigns a (subjective, though correct) probability of 1 to elements of ω that have been realized and are known with certainty at decision-making time.

Next consider a research project, RP_1 in which agronomic trials are run, data is analyzed, etc. The set of all conceivable signals from RP_1 is Y_1. Roughly stated, the aim of a research procedure is to provide the decision maker an update to his/her prior beliefs about the state of the world, $v_0(\omega|y_0)$, to make them more closely resemble $g(\omega)$, the true pdf on the states of nature. The decision maker uses a generic signal y_1 from Y_1 to update his/her beliefs according to Bayes' theorem, developing a new system of beliefs, called $v_1(\omega|y_1)$, about the probabilities of states of nature given signals have been sent from RP_1:

$$v_1(\omega|y_1) = \frac{v_0(y_1|\omega)v_0(\omega|y_0)}{\int_\Omega v_0(y_1|\hat{\omega})v_0(\hat{\omega}|y_0)d\hat{\omega}}. \quad (1)$$

It may be that after RP_1 is complete, a new research project, RP_2 is conducted. Depending on which set of results $y_2 \in Y_2$ comes out of RP_2, the research program will create another information structure with noise, $v_2(y_2|\omega)$, and again the decision maker will update his/her beliefs using Bayes' theorem:

$$v_2(\omega|y_2) = \frac{v_1(y_2|\omega)v_1(\omega|y_1)}{\int_\Omega v_1(y_2|\hat{\omega})v_1(\hat{\omega}|y_1)d\hat{\omega}}. \quad (2)$$

Continuing with this dynamic process, beliefs in a year t are updated from those in year $t-1$ using Bayes' theorem:

$$v_t(\omega|y_t) = \frac{v_{t-1}(y_t|\omega)v_{t-1}(\omega|y_{t-1})}{\int_\Omega v_{t-1}(y_t|\hat{\omega})v_{t-1}(\hat{\omega}|y_{t-1})d\hat{\omega}}. \quad (3)$$

Since the data set from RP_t contains all the data from RP_{t-1}, it is generally to be expected that analysis of RP_t will provide better estimates of the state of the world, and ultimately of management recommendations. But, for example, if the random influences on yield in year t are sufficiently unusual, then the new data may actually make the researcher's estimate of the true state of nature, and the recommendation for management strategies, worse. Of course, as more data are collected, the probability of continuing to draw random elements from the tails of their distributions becomes more remote. In our particular simulations, conducting additional field trials always generated data that improved management strategies.

Optimal Choices Conditional on Information

Because vector $\mathbf{w}$ includes both the market prices of hired inputs and the per-unit opportunity costs of owned inputs, then $pf(\mathbf{x}) - \mathbf{wx}$ are the producer's (annual) returns to owned inputs, which we will call "economic profits," for short.

Suppose a farmer has received a signal y. Then, with beliefs $v(\omega|y)$, if s(he) is developing a URA strategy and receives no additional information, (s)he will maximize her/his (subjectively) expected economic profits on the field, while being constrained to choose the same management plan on every plot. S(he) chooses,

$$X^{U^*}(v,y) = \begin{bmatrix} \mathbf{x}_1^{U^*}(v,y) \\ \vdots \\ \mathbf{x}_I^{U^*}(v,y) \end{bmatrix} = \begin{bmatrix} \mathbf{x}^{U^*}(v,y) \\ \vdots \\ \mathbf{x}^{U^*}(v,y) \end{bmatrix} = \operatorname*{argmax}_{\mathbf{x}_1,\ldots,\mathbf{x}_I} \sum_{i=1}^{I} \int_{\Omega} \Big(p[f(\mathbf{x}_i, \mathbf{c}_i, \mathbf{z}) + \varepsilon_i] - \mathbf{w}\mathbf{x_i} \Big) v(\omega|y) d\omega$$
$$\text{s.t.} \quad \mathbf{x}_1 = \cdots = \mathbf{x}_I. \tag{4}$$

The solution to equation (4) is the optimal uniform management map given the information structure (v, y). The realized economic profits associated with the optimal choices in year t are,

$$\Phi(\omega_t, X^{U^*}(v,y)) = \sum_{i=1}^{I} \Big[p_t \cdot f(\mathbf{x}_t^{U^*}(v,y), \mathbf{c}_i, \mathbf{z}_t) - \mathbf{w}_t \cdot \mathbf{x}_t^{U^*}(v,y) - C^{URA} \Big], \tag{5}$$

where C^{URA} is the per-plot cost of hiring custom uniform application of fertilizer (which does not include the price of the fertilizer itself).

The decision maker developing a VRA strategy with beliefs $v(y|\omega)$ maximizes his/her (subjectively) expected net revenues on the field, while not being constrained to choose the same management plan on every plot. (S)he chooses,

$$X_V^*(v,y) = \begin{bmatrix} \mathbf{x}_1^{V^*}(v,y) \\ \vdots \\ \mathbf{x}_I^{V^*}(v,y) \end{bmatrix} = \underset{\mathbf{x}_1,\ldots,\mathbf{x}_I}{\operatorname{argmax}} \sum_{i=1}^{I} \int_{\Omega} \Big(p[f(\mathbf{x}_i, \mathbf{c}_\mathbf{i}, \mathbf{z}) + \varepsilon_i] - \mathbf{w}\mathbf{x}_\mathbf{i} \Big) v(\omega|y) d\omega. \tag{6}$$

The realized economic profit associated with the optimal choices in year t is,

$$\Phi(\omega_t, X_V^*(v,y)) = \sum_{i=1}^{I} \Big[p_t \cdot f(\mathbf{x}_i^{V^*}(v,y), \mathbf{c}_\mathbf{i}, \mathbf{z}_t) - \mathbf{w}_t \cdot \mathbf{x}_i^{V^*}(v,y) - C^{VRA} \Big], \tag{7}$$

where C^{VRA} is the per-plot cost of hiring custom variable rate application of fertilizer (which does not include the price of the fertilizer itself).

The change in realized economic profits in year t due to using VRA instead of URA is:

$$V_{VRA}(\omega_t, v, y) = \Phi(\omega_t, X_V^*(v,y)) - \Phi(\omega_t, X_U^*(v,y)). \tag{8}$$

Annual Gross Value of Information (v,y) *in Year t*

The farmer currently possesses information (v_0, y_0). Given the information, the optimal URA and VRA strategies are $X^{U^*}(v_0, y_0)$ and $X^{V^*}(v_0, y_0)$. Let X_0 denote the farmer's current practice. We define the annual gross realized value of new information (v,y) conditional on strategy as the difference in year t's realized economic profits when having information (v,y) as compared to when having information (v_0, y_0) (the information available before the farmer receives (v,y)). Note this value does not consider the cost of obtaining the information. The realized values of having information (v,y) in year t when making decisions in year t for URA and VRA are,

$$V_{info}(v, y, \omega_t, URA) = \Phi(\omega_t, X^{U^*}(v, y)) - \Phi(\omega_t, X_0), \tag{9}$$

$$V_{info}(v, y, \omega_t, VRA) = \Phi(\omega_t, X^{V^*}(v, y)) - \Phi(\omega_t, X_0). \tag{10}$$

On-farm Field Trials

A *field trial* is a particular management map:

$$X^{FT} = \begin{bmatrix} \mathbf{x}_1^{FT} \\ \vdots \\ \mathbf{x}_I^{FT} \end{bmatrix}, \tag{11}$$

which is chosen by the researcher (for example, according to statistical protocols). Various aspects of management, including input application rates, product brands, and application timing can all be considered in this framework. Field trials can be run over a series of years, and the amount of information gleaned from trials depends on the number of years of experimentation.

Economic Profits during Field Trials

Consider a field trial in year t. The direct costs of running it involves the original "set-up" costs, C^S, of experimental design and in training the farmer to implement the trial (both assumed to be carried out in year 0). Also, in any year t in which the trial is run, C_t^{VRA} is paid (per plot) to hire the variable rate equipment and services necessary to implement the experiments. C_t^F denotes the year's cost to the farmer of the extra effort and time it takes to confer with researchers, etc, in order to implement the experiment's design.

Given that the realized state of nature is ω_t, since the crop produced during the trial can be sold, realized economic profits when running a field trial in year t are,

$$\Phi(\omega_t, X_t^{FT}) = \sum_{i=1}^{I} \left[p_t(f(\mathbf{x}_{it}^{FT}, \mathbf{c}_i, \mathbf{z}_t) + \varepsilon_{it}) - \mathbf{w}_t \mathbf{x}_{it}^{FT} - (C_t^{VRA}) \right] - C_t^F. \tag{12}$$

The present value of stream of economic profits stream from L years of field trials is,

$$\Pi^{FT}(L) = \sum_{t=1}^{L} \frac{1}{(1+r)^t} \Phi(\omega_t, X_t^{FT}) - C^S, \tag{13}$$

where the set-up cost C^S is paid at $t = 0$. The opportunity cost of running the on-farm experiment in year t is $\Phi(\omega_t, X_0)$, the economic profits the farmer could obtain if s(he) were to follow her/his subjective optimal N application rates when s(he) only has the status-quo information structure (v_0, y_0). Thus, the cost of running an experiment in year t can be defined as,

$$C^{FT}(\omega_t) = \Phi(\omega_t, X_0) - \Phi(\omega_t, X_t^{FT}). \tag{14}$$

Economic Profits after Experiments

For $j = 1, \dots, T-L, \omega_{L+j} \in \Omega$ describes the state of the world that is realized by the end of the growing season in year $L+j$. (ω_{L+j} might include a weather outcome, $\mathbf{z}_{L+j}$, price outcomes p_{L+j} and $\mathbf{w}_{L+j}$ and the realized values of the sites' production disturbances, $\varepsilon_{1,L+j}, \dots, \varepsilon_{I,L+j}$.) From equation (5), the realized economic profits in year $t \in [L+1, \dots, T]$ given the information from L years of experiments are $\Phi(\omega_t, X_t^{U^*}(v_L, y_L))$. The present value of the realized economic profits stream to the decision maker operating in year $L+1$ through T with a URA strategy under information structure (v_L, y_L) is shown in (15). Similarly, the present value of the realized economic profits stream using a VRA strategy is shown in (16).

$$\Pi^{post}(v_L, y_L, URA) = \sum_{t=L+1}^{T} \frac{1}{(1+r)^t} \Phi(\omega_t, X_t^{U^*}(v_L, y_L)), \tag{15}$$

$$\Pi^{post}(v_L, y_L, VRA) = \sum_{t=L+1}^{T} \frac{1}{(1+r)^t} \Phi(\omega_t, X_t^{V^*}(v_L, y_L)). \tag{16}$$

Present Values of the Streams of Realized Economic Profits over All Years

Summing equations (13) and (15) calculates the present value of the economic profits stream from running L years of experiments and then using the information learned to manage the field using URA:

$$\Pi(L, URA) = \Pi^{FT}(L) + \Pi^{post}(y_L, v_L, URA). \tag{17}$$

Similarly, summing (13) and (16), calculates the present value of the economic profits stream from running L years of experiments and then using the information learned to manage the field using VRA:

$$\Pi(L,VRA) = \Pi^{FT}(L) + \Pi^{post}(y_L, v_L, VRA). \tag{18}$$

If the farmer does not run on-farm experiments and continues with his/her subjectively optimal URA strategy with information (v_0, y_0), then the present value of the resulting stream of economic profits is,

$$\Pi(0,URA) = \sum_{t=1}^{T} \frac{1}{(1+r)^t} \cdot \Phi(\omega_t, X_0). \tag{19}$$

The Net Value of On-farm Field Trials

The net value of L years of experiments when the post-experiment strategy is to use URA is the difference in the present value of the profit stream when the experiments are run, as compared to when no experiments are run:

$$\Pi^{ft}(L,URA) = \Pi(L,URA) - \Pi(0,URA). \tag{20}$$

A little arithmetic shows that the net value of on-farm trials when post-trial strategy is URA equals the value of the information generated by the field trials, less the cost of those trials:

$$
\begin{aligned}
\Pi^{ft}(L,URA) =& \Pi(L,URA) - \Pi(0,URA) \\
=& \sum_{t=1}^{L} \Phi(\omega_t, X_t^{FT}) + \sum_{t=L+1}^{T} \Phi(\omega_t, X^{U^*}(v_L,y_L) - \sum_{t=1}^{T} \Phi(\omega_t, X_0) \\
=& \sum_{t=1}^{L} \Big[\Phi(\omega_t, X_t^{FT}) - \Phi(\omega_t, X_0)\Big] + \sum_{t=L+1}^{T} \Big[\Phi(\omega_t, X^{U^*}(v_L,y_L)) - \Phi(\omega_t, X_0)\Big] \\
=& -\sum_{t=1}^{L} C^{FT}(\omega_t) + \sum_{t=L+1}^{T} V_{info}(v_L, y_L, \omega_t, URA).
\end{aligned}
\tag{21}
$$

In the first L years, the farmer "loses" $\Phi(\omega_t, X_0) - \Phi(\omega_t, X_t^{FT})$ each year because experimental rates are applied instead of the subjective optimal rates given (v_0, y_0).[5] This is the cost of obtaining information in year t. However, after the L years of experiments, the farmer gains $\Phi(\omega_t, X^{V^*}(v_L, y_L)) - \Phi(\omega_t, X_0)$ each year by refining her/his N application strategy based on information (v_L, y_L) generated during the experimental phase. This term is the value of information $V_{info}(v_L, y_L, \omega_t, URA)$ defined earlier.

The net value of conducting L years of field trials and then employing an optimal URA strategy after the trial years (denoted as $\Pi^{ft}(L,VRA)$) is,

$$
\Pi^{ft}(L,VRA) = \Pi(L,VRA) - \Pi(0,URA). \tag{22}
$$

Similar to equation (21), we can write $\Pi^{ft}(L,VRA)$ as,

$$
\begin{aligned}
\Pi^{ft}(L,VRA) =& \Big(\Pi(L,VRA) - \Pi(L,URA)\Big) + \Big(\Pi(L,URA) - \Pi(0,URA)\Big) \\
=& \sum_{t=L+1}^{T} \Big[\Phi(\omega_t, X^{V^*}(v_L,y_L)) - \Phi(\omega_t, X^{U^*}(v_L,y_L))\Big] + \Pi^{ft}(L,URA) \\
=& \sum_{t=L+1}^{T} V_{VRA}(\omega_t, v_L, y_L) - \sum_{t=1}^{L} C^{FT}(\omega_t) + \sum_{t=L+1}^{T} V_{info}(v_L, y_L, \omega_t, URA).
\end{aligned}
\tag{23}
$$

The first term is the value of VRA compared to URA given information (v_L, y_L), which we denote $\Pi_{ft}^{VRA}(v_L, y_L)$. Thus, the net value of on-farm field trials using VRA is the summation of the net value of VRA compared to URA, and the value of the information using URA, less the cost of generating information (v_L, y_L).

Optimal Experiment Length and Maximized Net Value of On-farm Field Trials

After calculating the net present value of on-farm field trials using URA and VRA conditional on L years of experiments, the optimal number of years of experimentation can be found easily for both URA and VRA. Let L_U^* and L_V^* denote the optimal number of years of experiments for URA and VRA. Then the maximum obtainable present value of the income stream is shown in (24), and the optimal experiment length is either L^{V^*} or L^{U^*}, depending on which solves the maximization problem:

$$\Pi^{ft} = max\Big\{\Pi^{ft}(L_V^*, VRA), \Pi^{ft}(L_U^*, URA)\Big\}. \tag{24}$$

Value of Information

Next, we examine how valuable (y_L, v_L) (information equivalent to that generated by running L years of experiments on the decision maker's farm) would be if it were given to the farmer at $t = 0$. The expected value of this term is the farmer's willingness to pay for receiving information (v_L, y_L) at the beginning of the first year.

Using equation (10), the value of information (y_L, v_L) in year t if the URA strategy is used is $V_{info}(v_L, y_L, \omega_t, URA) = \Phi(\omega_t, X^{U^*}(v_L, y_L)) - \Phi(\omega_t, X^{U^*}(v_0, y_0))$. Thus, the value of information (y_L, v_L) over the T-year period is simply the discounted sum of the value of information:

$$\Pi^{info}(v_L, y_L, URA) = \sum_{t=1}^{T} \frac{1}{(1+r)^t} \cdot \left[\Phi(\omega_t, X^{U^*}(v_L, y_L)) - \Phi(\omega_t, X_0)\right]. \tag{25}$$

Analogously, the value of information (y_L, v_L) over the T-year period under the VRA strategy is,

$$\Pi^{info}(v_L, y_L, VRA) = \sum_{t=1}^{T} \frac{1}{(1+r)^t} \cdot \left[\Phi(\omega_t, X^{V^*}(v_L, y_L)) - \Phi(\omega_t, X_0)\right]. \tag{26}$$

The value of VRA conditional on the free provision of information (v_L, y_L) is,

$$\Pi_{fi}^{VRA}(v_L, y_L) = \sum_{t=1}^{T} \frac{1}{(1+r)^t} \cdot \left[\Phi(v_L, y_L, \omega_t, VRA) - \Phi(v_L, y_L, \omega_t, URA)\right]. \tag{27}$$

Monte Carlo Simulations

We assume henceforth that management practices other than nitrogen fertilizer application rates are held constant.[6] This common procedure equates a change in accounting profits (the change in payments to owned inputs, which we call simply "profits") with the changes in economic profits. Suppressing the notation for the other inputs, we let N_{it} denote the nitrogen fertilizer rate applied to plot i in year t (instead of x_{it}). Further, we denote the vector of N rates for the entire field as $\mathbf{N}_t = (N_{1t}, \ldots, N_{It})$. For example, $X^{U^*}(v_L, y_L)$ and $X^{V^*}(v_L, y_L)$ are denoted $\mathbf{N}^{U^*}(v_L, y_L)$ and $\mathbf{N}^{V^*}(v_L, y_L)$.

We assume the size of a field is 80 acres, which consists of 320 plots. In recognition that seed technology and thus yield response can change over time, we use $T = 10$ years as the duration of the economic problem. Changing the duration would shift the optimal number of years of experiments, but most of the economic intuition obtained with $T = 10$

remains intact with larger values of T. We assume that throughout the 10-year period, the prices of corn and nitrogen fertilizer are fixed at $p = \$3.90$/bu and $w = \$0.50$/lb. While in reality prices fluctuate substantially over time, allowing for random prices would simply make our simulations more cumbersome without providing appreciable economic insights. We also assume that when no experiments are conducted, the farmer applies the MRTN rate of 156 lb/acre everywhere on the field, which is located in central Illinois, employing a corn-after-corn rotation. In terms of our formal notation, $\mathbf{N}_i^{U^*}(y_0, v_0) = \mathbf{N}_i^{V^*}(y_0, v_0) = 156$ lb/acre, for all i.[7] Custom fertilizer application prices vary in the U.S. Corn Belt. From a survey of Iowa custom applicators, Plastina, Johanns, and Erwin (2016) report an average charge of \$6.65/acre for uniform-rate spraying of liquid fertilizer. For those same services, Miller (2013) reports an average charge of \$6.14/acre from a survey of Indiana custom applicators, Stein (2014) reports an average charge of \$7.60/acre by custom applicators in Michigan, and Halich (2016) reports an average \$6.00/acre in Kentucky. Halich (2016) also reports that custom applicators in Kentucky charged \$2.00/acre more to variably apply (instead of uniformly apply) dry fertilizer. We assume in our simulations that producers pay $C^{URA} = \$7.00$/acre for uniform custom application of liquid fertilizer, and pay $C^{VRA} = \$9.00$/acre for variable rate application. In current on-farm experiments, colleagues of the authors are conducting one two-hour meeting in the first year of the experiments with participating farmers, to inform them of the basic experimental procedures. Including travel time, we assume an opportunity cost of \$200 to participate in the meeting. The researchers have found that in the first year of experiments, farmer have fairly frequent questions about experimental procedures, and the research project pays farmers a lump sum of \$500 in the first year as compensation for their communication efforts. So for year 1 of an experiment, we assume $C^F = \$700$. But in subsequent years such consultation becomes less necessary. For year two, we assume $C^F = \$400$, for year three $C^F = \$200$, and for years four through ten, $C^F = \$100$. All the other purchased input costs are assumed to be \$435/acre, following

Table 2 of Schnitkey (2015).[8]

We base our data-generating process on an empirical study of corn yield response to N fertilizer rates and field characteristics, reported in Ruffo et al. (2006) and Bullock et al. (2009), who generated data conducting on-farm randomized agronomic experiments in fields in Champaign County, Illinois, using PAT in the manner described in the second section. On that same field, the researchers measured two spatially variable field characteristics, called *ISNT* (Illinois Soil Nitrogen Test) and *SPI* (Stream Power Index). Bullock et al. (2009) identified the Illinois Soil Nitrogen Test (*ISNT*) as the key factor affecting the marginal productivity of nitrogen. *ISNT* provides an estimate of the nitrogen available in the soil and was developed by Khan, Mulvaney, and Hoeft (2001). The greater the value of *ISNT*, the smaller will be the marginal impact of N fertilizer. This is reflected by the interaction terms of I with N and N^2. In our simulations, *ISNT* is the key variable that varies within a field that makes each subplot respond differently to N fertilizer applied.

We ran all the simulations using *R* (R Core Team, 2016), and the R programs are available in an on-line supplementary appendix.

Simulating Spatially Specific Corn Yield Responses

Bullock et al. (2009) reported the following estimate of a corn yield response function:

$$\begin{aligned} Y_t =& \beta_0 + \beta_N \cdot N_t + \beta_I \cdot I + \beta_S \cdot S + \beta_{N^2} \cdot N_t^2 \\ &+ \beta_{NI} \cdot N_t \cdot I + \beta_{NM} \cdot N_t \cdot M_t + \beta_{NS} \cdot N_t \cdot S + \beta_{N^2 I} \cdot N_t^2 \cdot I \end{aligned} \tag{28}$$

where, $\beta_0 = 5524$, $\beta_N = 154.97$, $\beta_I = 52.33$, $\beta_S = -194.7$, $\beta_{N^2} = -0.4461$, $\beta_{NI} = -0.4866$, $\beta_{NM} = 0.1624$, $\beta_{NS} = 0.7537$, and $\beta_{N^2 I} = 0.001318$. In (28) yield Y_t is measured in kg/ha. We make appropriate conversions of units and present results on a per-acre basis to be consistent throughout this article. In our simulations, we use this function as the deterministic part of the meta yield response function, $f(\mathbf{x},\mathbf{c},\mathbf{z})$, where $\mathbf{x} = N$, the nitrogen fertilizer application rate, $\mathbf{c} = (S, I)$, the *SPI* and *ISNT* values, and $\mathbf{z} = M$ (May rainfall). The estimated coefficients b_{NI} and b_{NM} were both significantly different from zero, but the impact of the *SPI* characteristic on the marginal product of N was negligible in comparison to the impact of *ISNT* on the marginal product of N. This causes *ISNT* to play a much bigger role in the determination of spatially specific economically optimal N rates in our simulations.

We generated the field's *ISNT* and *SPI* maps using the following spatial autoregressive process:

$$Q = \rho_q W Q + e \tag{29}$$

$$e \sim N(\mu_q(1-\rho_q), \sigma_q^2), \tag{30}$$

where ρ_q is the spatial autoregressive parameter, W is the spatial weights matrix, and μ_q and σ_q^2 are the expected value and variance of Q. The spatial weights matrix W is specified as the row-normalized version of a matrix whose (i, j) element is $1/d_{i,j}^2$, where $d_{i,j}$ is the distance between the centroids of ith and jth plots.

Bullock et al. (2009) also found that precipitation in May enhances the marginal productivity of N fertilizer, as indicated by the positive coefficient on the interaction term between N and M. Presumably, this is because May precipitation encourages nitrogen leaching and run-off, and consequently applied N fertilizer has a greater marginal impacts on yield. Unfortunately, the coefficient estimate on the interaction between N and M seems unreliable. Figure 2 shows the optimal N rate conditioned on the value of *ISNT* for different values of May precipitation. For 30, 60 and 90 mm of precipitation, the optimal N rate decreases as the value of *ISNT* increases, consistent with agronomic theory. But when May precipitation is 120*mm*, the optimal N rate becomes greater as *ISNT* increases. As Bullock et al. (2009) used only two years of experimental data, they did not have enough variation or range of May precipitation to reliably estimate its interaction with N fertilizer. In this study, we assume that May precipitation takes a constant value of 90.

As do Kapoor, Kelejian, and Prucha (2007) (KKP), we specify the error term u_t, where the error terms are stacked vertically. Mathematically,

$$u_t = \lambda W u_t + v \tag{31}$$

$$v = \mu + \varepsilon_t \tag{32}$$

$$\mu \sim N(0, \sigma_\mu^2 \cdot I_M) \tag{33}$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2 \cdot I_M), \tag{34}$$

where μ is 320×1 plot-level time invariant characteristics, ε_t is 320×1 error term that varies over both plot and time, and W is the same spatial weight matrix used in equation (30). In this model, both plot-level characteristics and idiosyncratic error terms are spatially correlated.[9]

Dynamic Field Trial Design

We simulate a farmer running annual field trials in which each of the five experimental N rates is applied on 64 of the 320 plots, with the locations of plots receiving the various N rates chosen at random. For $L = 0, 1, \ldots, 9$, we run simulations in which trials are conducted for L years, and in which the information generated by the series of trials is then used to manage the field in years $L+1, \ldots, T = 10$.

In the first-year trials, the five N rates are centered around the central Illinois MRTN rate of 156 lb/acre, and are 111, 134, 156, 178, and 201 lb/acre. Yields are determined on a plot i by substituting one of the five N rate values and the values of SPI_i and $ISNT_i$ into the model's yield response function in (28), and then adding to that a drawing of a yield disturbance term u_{it}. In each Monte Carlo run, this generates a data set with 320 yield, N rate, $ISNT$, and SPI values. We assume that the functional form of the true yield response function is known, but that its coefficients are not known by the researcher or decision maker. We estimate a spatial error model (Anselin, 2013) to obtain point estimates of the coefficients in (28). We use these point estimates as the first-year signal, y_1, about the true state of nature, and create an information structure by assuming that this signal assigns a subjective probability of 1 that those estimates equal the true values of the coefficients. We then solve the profit maximization problems in (4) to estimate $N^{U^*}(v_1, y_1)$, the profit-maximizing uniform N application rate given the information available after the first-year trial. Similarly, we solve the maximization problem in (6) to estimate $N_i^{V^*}(v_1, y_1)$ for all i.

We carried out similar econometric procedures in the second year of trials (for simulations in which $L \geq 2$), except we changed the values of the five experimental N rates to center them around the point estimate $N^{U^*}(v_1, y_1)$ computed from the data in the first-year trial. When we assumed more than one year of trials, then in each Monte Carlo run year 2's trial generated an additional data set, and in each Monte Carlo run we combined the data from the second-year trial with the data from the first-year trial, re-estimated the yield response function coefficients, and then used those to solve for $N^{U^*}(v_2, y_2)$ and $N_i^{V^*}(v_2, y_2)$ for all i. Unlike after the first trial year, we use the panel data spatial error model (hereafter, PSE) developed in KKP to estimate the yield response function.[10] In the presence of spatial dependence in the error term, in general OLS is not efficient. But the PSE is a GLS estimator that takes into account the spatial dependence of both plot-level effects and idiosyncratic errors specified above. For each year up to year L, we continued this algorithm of centering in the year-t experimental N rates around $N^{U^*}(v_{t-1}, y_{t-1})$, and used the data to estimate the response function coefficients, ultimately obtaining estimates $N^{U^*}(v_L, y_L)$ and $N_i^{V^*}(v_L, y_L)$ for all i.

Results and Discussions

Cumulative Information and Estimation Accuracy

Figure 3 shows for VRA (left panel) and URA (right panel), and for differing numbers of years of on-farm trials, the distribution of the ratio of estimated optimal N rates to the true optimal N rates. It is clear from the figures that as more data are collected, the density mass is more tightly centered around 1, indicating improved estimation of the yield-response function for both cases. The question is the extent to which the improvements in estimation accuracy lead to profit gains. Figure 4 shows the expected annual profit of a farmer in the absence of fertilizer application costs (C^{URA} and C^{VRA}) using the estimated optimal N rates (site-specific for VRA and uniform for URA) given s(he) has available various numbers of years of on-farm trial data. For both VRA and URA, data from on-farm trials exhibit a sharply diminishing marginal return to annual profit. For example, running three more years of on-farm trials after five years of experiments results in a negligible annual profit gain even though the additional three years of data improve statistical efficiency as shown in figure 3.

The time patterns of the values of information for VRA and URA are different in an important way. When the estimated optimal N rates are based on only one year of data, URA outperforms VRA by a wide margin. This occurs because both URA and VRA are based on *estimated* yield-response functions. It is possible that estimated optimal N rates under VRA deviate substantially from the true optimal N rates, especially when data is relatively scarce. On the other hand, URA is robust to this type of loss because these deviations tend to be averaged out. An additional year of data improves the annual profitability of VRA and URA by \$5.30/acre and \$1.26/acre. Consequently, once two years of data are available, VRA outperforms URA. Any additional years of data provide almost no value to

URA. Indeed, the third year of data enhances the annual profit of URA only by \$0.11/acre. The third year of data raises annual profit under VRA by only \$0.712/acre. As more data are accumulated, the gap between the value of VRA and URA slowly widens.

Once application costs, C^{URA} and C^{VRA}, are taken into account, the relative performance of URA and VRA strategies reverses, as shown in figure 5. That is, the additional revenues brought by VRA compared to those from URA are not high enough to compensate for the additional application costs VRA entails.

Optimal Duration of Field Trials

On-farm trials have trade-offs. While trials are run, information is generated to be used in the later periods to improve N application, whether under VRA or URA, which in turn can result in higher annual profits. At the same time, in the trial, the farmer must apply a range of N rates, which in general will not equal the site-specific economically optimal rates. Thus, conducting trials may lead to a substantial profit losses. Therefore, for on-farm trials to be of net benefit, the value recouped in the later periods by improved N application ($\sum_{t=L+1}^{T} V_{info}(v_L, y_L, \omega_t, S)$) has to outweigh the loss generated in running experiments ($\sum_{t=1}^{L} C^{FT}(\omega_t)$). If the farmer runs too many years of on-farm trials, fewer years will remain to practice refined N application, and too large a loss in profit will be suffered in the experimental phase. On the other hand, if the farmer runs too few years of experiments, refinement in the subsequent N application might be compromised.

Figure 6 shows the means (over the Monte Carlo simulation runs) of the present value of the profit streams per acre (\$/acre) as the number of years of experimentation changes from zero to nine. The figure reports that if the post-experiment strategy is VRA, it is optimal to run on-farm agronomic experiments for 2 years before stopping experimentation and thenceforth using the information to farm site-specifically. That is, the marginal annual value of information obtained through on-farm trials exceeds the annual marginal cost of running trials for the first two years. As with VRA, if the post-experiment strategy is URA, it is still optimal to run two years of on-farm experiments before using the information in uniform rate management in the third and following years.

Figure 7 provides further insights into the optimal strategies for VRA and URA. The figure shows the time pattern of profits by strategy. Whether the post-experiment strategy will be VRA or URA in the first year, an on-farm trial is run. The resulting profit is lower than that of following the MRTN rate. For both VRA and URA as the post-trial strategies, in the second year of the trial experimental N rates are updated based on first-year results. Consequently, the profitability of the on-farm trial exceeds that of MRTN, reducing the opportunity cost of running on-farm trials. In the third year, for both URA and VRA as post-trial strategies, it is optimal to stop experimenting and implement a strategy URA and VRA based on the previous two years of experiments. From the third year to the last, the farmer makes significantly more than under the MRTN strategy each year, making agronomic experiments worthwhile to undertake.

The Net Value of On-farm Field Trials

The value of being able to run on-farm field trials before managing with strategies VRA and URA are presented in figure 8. The value of an on-farm trials is maximized when the optimal switching year is chosen: two years under VRA and URA. The maximum expected values of on-farm trials for VRA and URA are \$45.79/acre and \$57.13/acre.

As shown in equation (23), The net value of on-farm trials using VRA can be decomposed into two parts: the value of VRA compared to URA given the information (v_L, y_L), and the value of on-farm trials under URA. For the optimal URA strategy (two years of experiments), the value of VRA over URA ($\Pi_{ft}^{VRA}(2)$) is $-$\$11.34/acre, and the value of

URA ($\Pi^{ft}(L, URA)$) is \$57.13/acre. *That is, the value of precision agriculture technology does not lie in its ability to implement VRA. Rather, most of the value of the technology comes from the the improved management made possible by the information derived from the field trials the technology permits.* This value of PAT has not been recognized in previous papers because previous research simply compared profitabilities of URA and VRA assuming complete information about how yields respond to input management. Of course, the value of on-farm field trials depends critically on the farmer's default practice, which we have assumed to be the application of the MRTN rate (X_0). If X_0 is close enough to the true optimal N rates in the first place, the farmer would lose by running on-farm experiments, because experiments are costly. The farmer would lose $C^{FT}(\omega_t)$ every year while running experiments, and enjoy only very small gains in profit in subsequent years ($V_{info}(v_L, y_L, \omega_t, URA)$). If, on the other hand, X_0 is worse than MRTN, the farmer would see an even greater value of on-farm field trials than are seen in our simulations.

The Gross Value of Information

The gross values of information (v_L, y_L) for strategies VRA and URA (equations (25) and (26)) are presented in figure 9. Unlike the net value of on-farm field trials needed to generate the information, the gross value of information is monotonically increasing in the number of years of experiments, since we consider that the farmer bears none of the cost of generating the information. As can be seen in the figure, once four years of experimentation are conducted, additional years of experimentation generate little value (as is also seen in figure 5). For example, the gross values of information equivalent to that generated by running three years of trials (v_3, y_3), are $\Pi^{info}(v_3, y_3, URA) = \$89.35$/acre and $\Pi^{info}(v_3, y_3, VRA) = \$80.02$/acre. The value of VRA conditioned on the free provision of information (v_L, y_L), denoted $\Pi^{VRA}_{fi}(v_L, y_L)$, is simply the difference between the value of information using VRA as compared to that of using URA, as defined in equation (27). For example, for (v_3, y_3), $\Pi_{fi}^{VRA}(v_3, y_3) = -\$9.33$/acre. Indeed, for any L $(= 1, \dots, 9)$, $\Pi_{fi}^{VRA}(v_L, y_L) < \Pi_{fi}^{URA}(v_L, y_L)$ for this particular site. This means that farmers would have no incentive to hire custom variable-rate fertilizer application if information were free for the taking. (Presumably, if the farmer did not hire custom application, but instead invested in precision agriculture equipment and in learning how to operate it, any resultant management flexibility would not pay for the investment.) On the contrary, to farmers who do not possess the information, PAT is much more valuable, because it enables the farmer to run trials on their own fields. As shown earlier, the farmer can make \$57.13/acre more by running 2 years of agronomic experiments using PAT and then implementing URA subsequently. This difference comes about not because PAT can be used for variable rate management after the field trials are complete, but rather because PAT can be used to generate information in the trials. That information is valuable, even though once (s)he has it, it makes more sense for the farmer to choose to manage fertilizer uniformly rather than variably.

The Impacts of Field Size

As the amount of information collected in one year increases with field size, field size can have important impacts on the results. To examine the effect of field size, we also conducted MC simulations for 40- and 160-acre fields.

Figure 10 shows how accurate optimal N rate estimations are. As expected, given any number of years of experiments, increasing the field size resulted in more accurate estimation of optimal variable and uniform N rates. Consequently, annual profits per acre conditional on (v_L, y_L) increase with field size, as shown in figure 11. Moreover, since more information is collected in each year of experimentation, increasing field size lowers the optimal number of trial years. Figure 12 shows the optimal numbers of years of experiments before implementing a URA strategy are two, two, and one for 40, 80, and 160 acres. For VRA, the optimal numbers of years of experiments are three, two, and two for 40, 80, and 160 acres. Notice that the optimal numbers of years of experiments for URA are lower than for VRA when the size of field is 40 and 160. This is because the value of information is lower if URA is the post-experiments management strategy. Roughly speaking, optimal uniform management involves managing to the field's "average" response function rather than to its site-specific response functions, and this takes less information.

As field size increases, the increased annual profit immediately translates into increased value of on-farm field trials, as can be seen in figure 12. Table 1 shows the the value of on-farm field trials for each strategy when optimal years of experiments are chosen. For URA, on-farm field trials are about \$14/acre (30% increase) more valuable when they are implemented on a 160-acre field compared to a 40-acre field. The gap between the values of URA and VRA closes as the field size increases. This is because more information is generated on a larger field, which benefits the more information-intensive VRA strategy more than it does the URA strategy. However, even at 160 acres, URA outperforms VRA.

Conclusions

We have presented an economic theory of on-farm field trials. The theory follows Laffont (1989) to define the value of information from those trials. Unlike most previous studies, we have clearly distinguished between the values of information and precision agriculture technology. Using Monte Carlo simulations, we examined numerically the optimal length of experimentation and the value of information from such field trials.

In our simulations, on-farm field trials provided a significant profit boost over the default N application strategy, which we assumed to follow the Maximum Return to Nitrogen

(MRTN) approach currently recommended by several U.S. land grant university extension services. Most important, the profit boost came from improved knowledge about the yield response function, and not from simply employing a variable rate application strategy in place of a uniform rate strategy. The implication is that the total value of PAT may have been severely underestimated in previous studies that focused only on PAT's ability to make VRA feasible, but ignored the potential value of using PAT to conduct randomized trials.

The optimal length of on-farm experiments depended on the application technology used in years following the trials, and also on field size. For an 40-acre field, two years of experimentation were optimal if uniform rate application technology was to be used after the conclusion of experiments, and three years if precision agriculture technology was to be used. This is because VRA requires more information to implement properly. The optimal length of on-farm experiments decreased with field size because more information could be generated each year with a larger field.

As are all other studies of the PAT technology, the research we have reported is limited in that we looked at only one particular field. In the coming years, a research team of which we are part will conduct one hundred large-scale field trials on farmers' fields in Illinois, Kentucky, Nebraska, Argentina, and Uruguay. Using the data generated in these experiments, we will have much more generalizable findings on the value of on-farm field experiments and information generated. Finally, in the research here reported we only used one particular experimental design. Finding optimal dynamic experimental designs to make the most of experiments is an interesting and important topic for future research.

Notes

[1]We will use four key terms and acronyms in this article to describe to technologies and two management strategies that employ those technologies. The first technology is *precision agricultural technology* (PAT), by which we mean a suite of satellite-based technologies that permit the site-specific management of crops. (That is, we mean the equipment itself.) Variable rate fertilizer spreaders, GPS-linked yield monitors, and computer hardware and software used to manage such equipment are all elements of the PAT suite. The second technology is *conventional agricultural technology* (CAT), by which we mean agricultural machinery, such as fertilizer spreaders and planters used to apply inputs at uniform rates on whole fields. The first management strategy is *variable rate application* (VRA). A farmer uses PAT to implement VRA. The second management strategy is *uniform rate application* (URA). A farmer uses CAT to implement URA. We distinguish between PAT and VRA, and also between CAT and URA, because PAT can generate economic value in three ways. First, PAT can generate economic value when it is used to run field trials that produce information. Second, PAT can generate economic value when the farmer uses it to implement a variable rate strategy. The value of the strategy is different from the value of the technology, and we must distinguish between these two. Third, in some of our Monte Carlo simulations it is optimal for a farmer to use PAT to run field trials, but then to choose a uniform rate management strategy, URA, in years that follow the experiment. So, it is possible that PAT can be used to produce information, but not used after that information is generated. Thus, we have to distinguish between the value of the technology itself and value of managing with that technology given the information at hand.

[2]For example, Thrikawala et al. (1999), Wang et al. (2003), Lambert, Lowenberg-Deboer, and Malzer (2006), and Bullock et al. (2009) use agronomic field experiments data. Roberts et al. (2002) used yield-N data simulated using the Environmental Policy Integrated Climate (EPIC) crop growth model. Bullock, Lowenberg-DeBoer, and Swinton (2002) assumes agronomically sensible yield functions.

[3]The input vector x may consist of "flows" of inputs hired in markets and also of inputs owned by the decision maker, such as the producer's own land, labor, and human capital.

[4]Input prices in vector w may be either market prices, or shadow prices representing per-unit opportunity costs of owned inputs.

[5]It is possible that one actually gains while running field trials if X_0 is a very poor estimate of the optimal uniform management map.

[6]Assuming that uses of owned inputs are constant enables us to equate the change in accounting profits,

which is observable, with the change in returns to owned inputs, which is not. For example, let x_1 be an input hired in markets, and let x_2 be an owned input. Our welfare measure, "economic profits," would then be $pq - w_1x_1 - w_2x_2$. Because w_2 and x_2 are unobservable, economic profits are unobservable. On the other hand, payments to owned inputs are $pq - w_1x_1$. If w_2x_2 is constant, then the change in economic profits equals the change in accounting profits, and so becomes observable even though levels of economic profits are not observable.

[7]Note that since the MRTN provides no site-specific information, then the optimal management plan in every plot is the same, even if VRT were to be used.

[8]Other purchased costs are obtained by subtracting fertilizer costs and C^{URA} from the total non-land costs for the second column of table 2 in Schnitkey (2015).

[9]On the contrary, in the error process considered in Baltagi, Song, and Koh (2003), plot-level time invariant characteristics are not spatially correlated. In the context of agricultural production, KKP's error specification seems more appropriate. For example, some unobservable soil characteristics are clearly spatially dependent and are usually very slow to change over time. Thus, we chose KKP's specification.

[10]PSE was estimated using the splm packages in R (Millo and Piras, 2012). The PSE method first estimates the variance components of the error term using GMM, and then runs a feasible GLS estimation using the estimated variance covariance matrix of the composite error term.

Strategy	40 acres	80 acres	160 acres
URA	\$50.01	\$57.13	\$64.56
VRA	\$23.44	\$45.79	\$60.33

Table 1: Increase in the present value of maximized profits stream per acre by field size and post-trial strategy

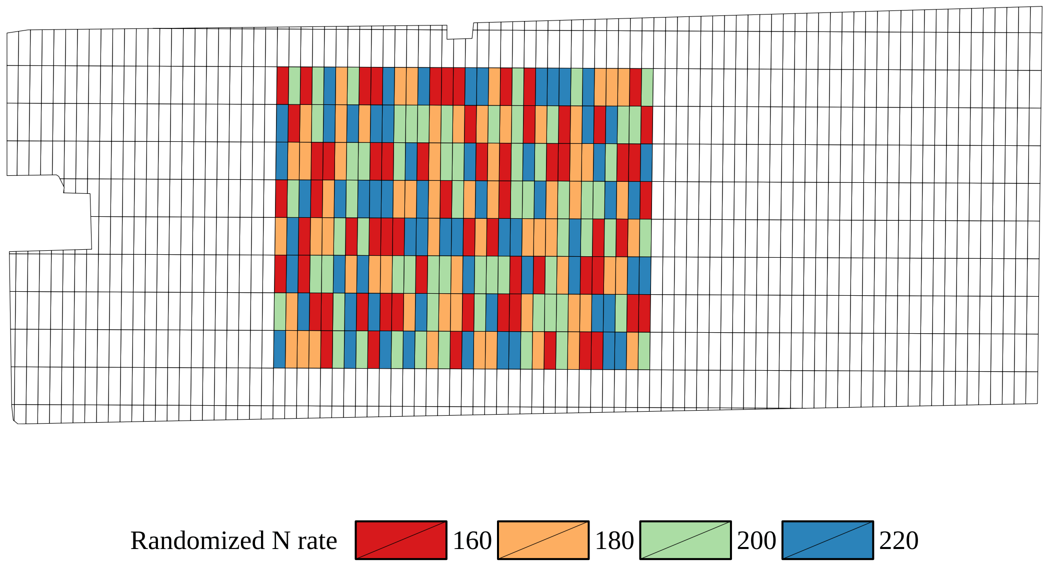

Figure 1: Randomized N rates ($/acre) at a field in Illinois

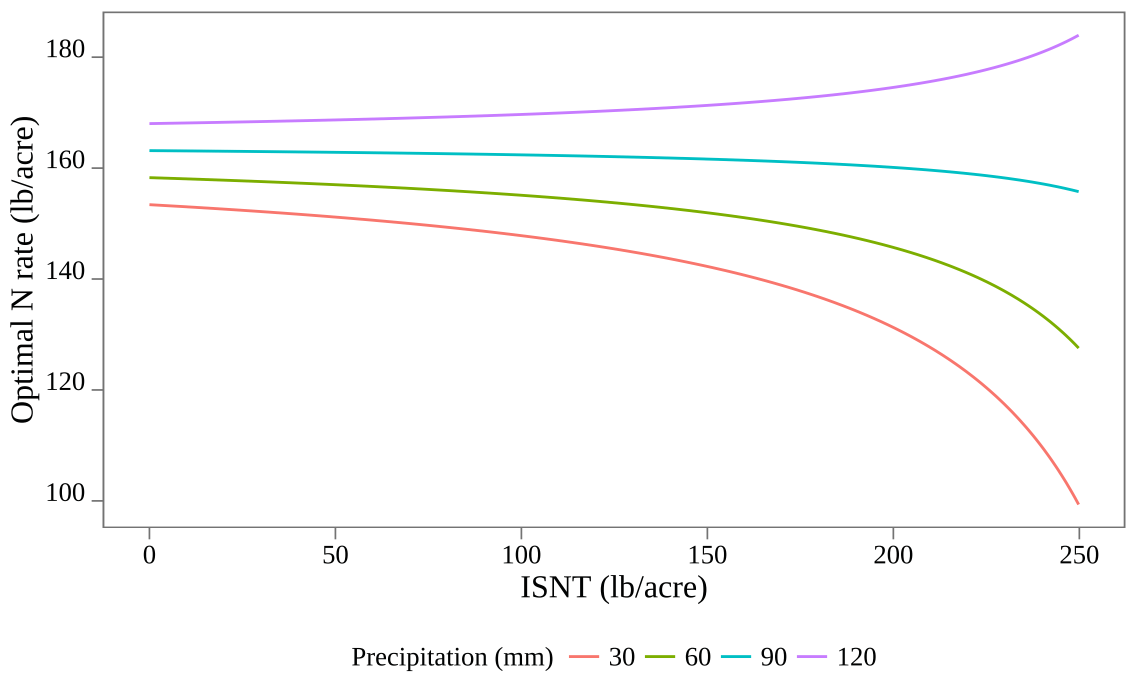

Figure 2: The impact of May precipitation on optimal N rate

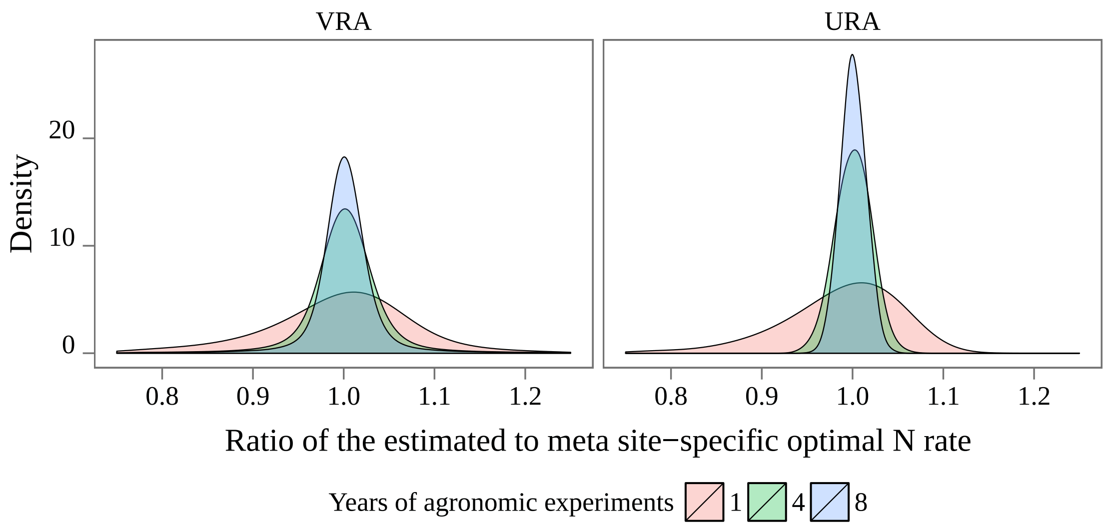

Figure 3: Convergence of estimated optimal N rates to the meta optimal N rates for VRA and URA

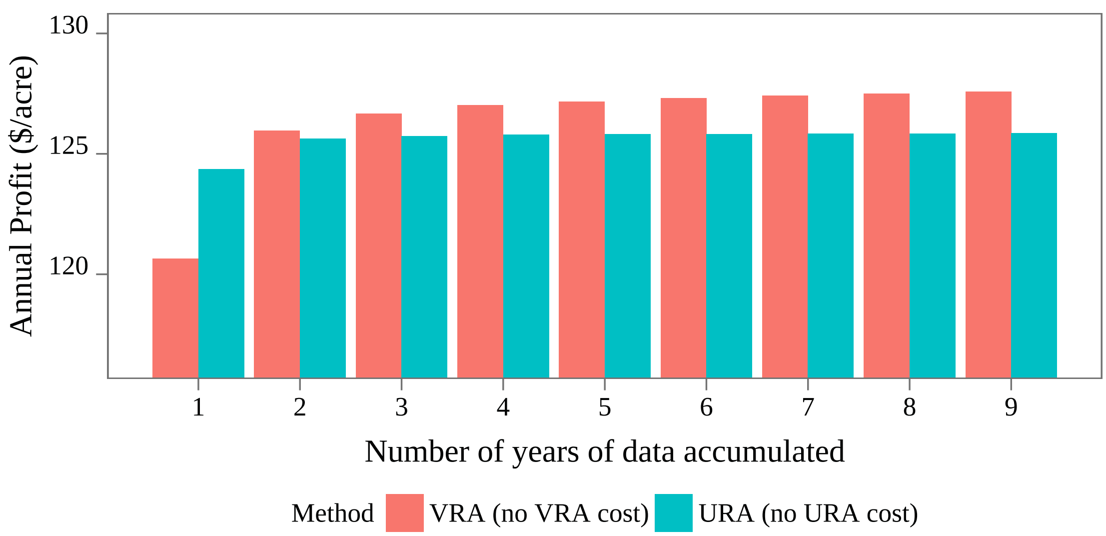

Figure 4: Profitability of VRA and URA before subtracting application costs

Note: URA and VRA represents $E[\Phi(X^{V^*}(v_L,y_L))]$ and $E[\Phi(X^{V^*}(v_L,y_L))]$.

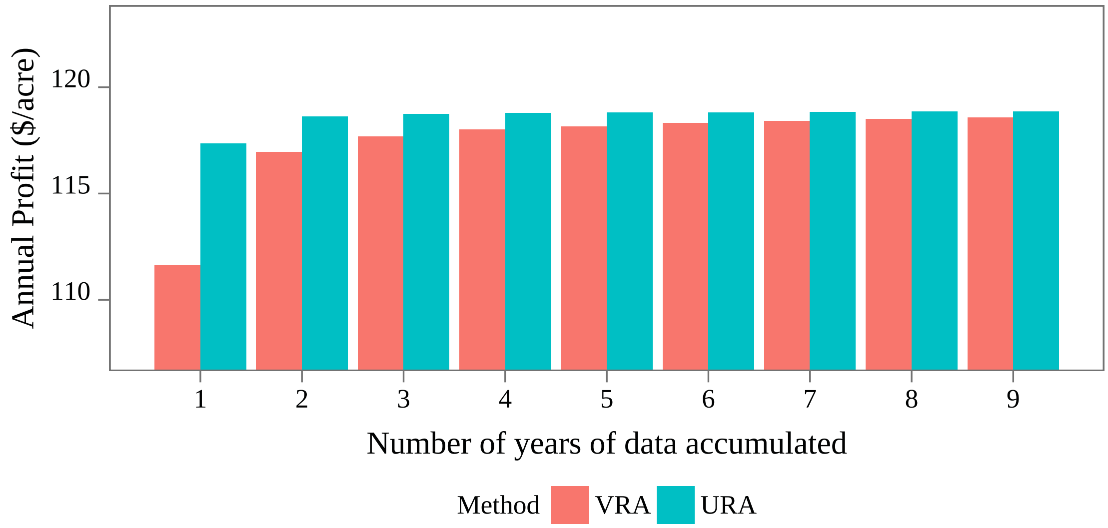

Figure 5: Profitability of VRA and URA

Note: URA and VRA represents $E[\Phi(X^{V^*}(v_L, y_L))]$ and $E[\Phi(X^{V^*}(v_L, y_L))]$.

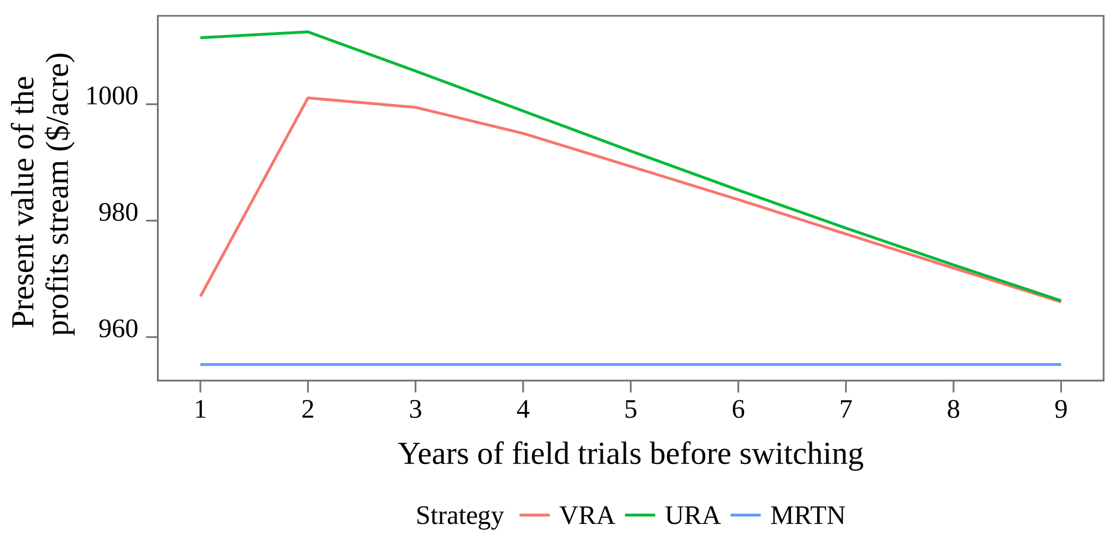

Figure 6: Present value of profit streams under VRA, URA, and MRTN

Note: $MRTN$, URA, and VRA represents $E[\Pi(0, URA)]$, $E[\Pi(L, URA)]$, and $E[\Pi(L, VRA)]$.

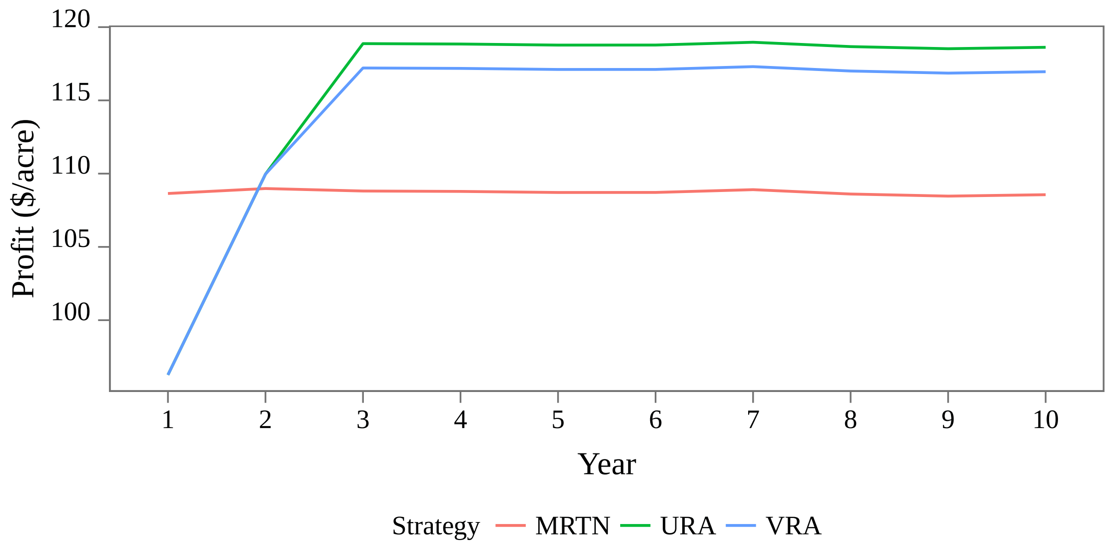

Figure 7: Mean annual profit in a 10-year period when optimal number of years of experiments are run

Note: *MRTN*, *URA*, and *VRA* represents $E[\pi(\omega_t, X^{U^*}(v_0, y_0))]$, $E[\pi(\omega_t, X^{U^*}(v_2, y_2))]$, and $E[\pi(\omega_t, X^{V^*}(v_3, y_3))]$ for each year.

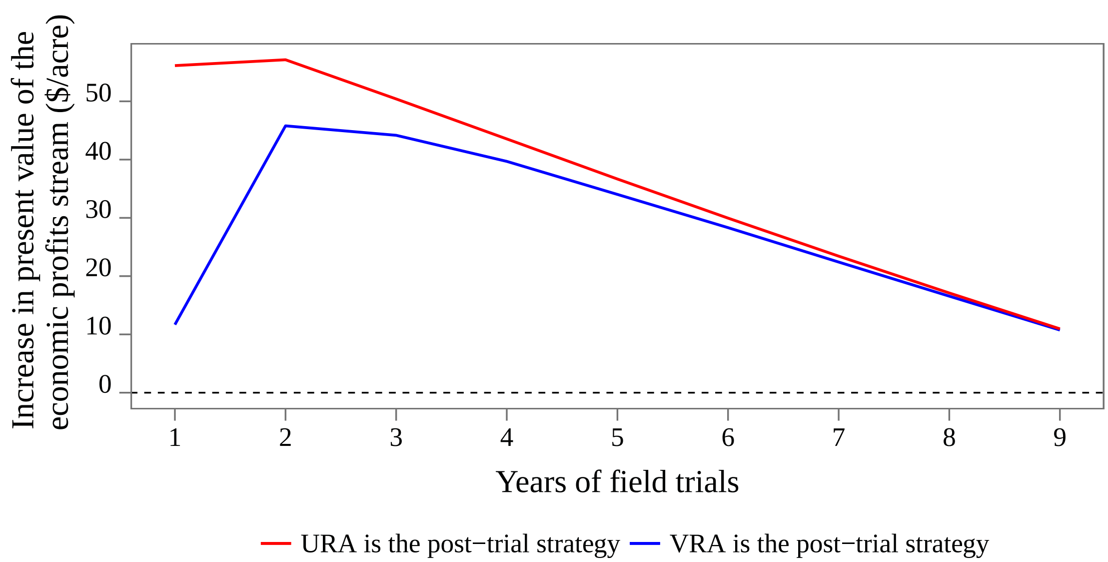

Figure 8: Increase in the present value of profits stream due to on-farm field trials

Note: Mathematically, *URA* and *VRA* represent $E[\Pi^{ft}(L, URA)]$ and $E[\Pi^{ft}(L, VRA)]$.

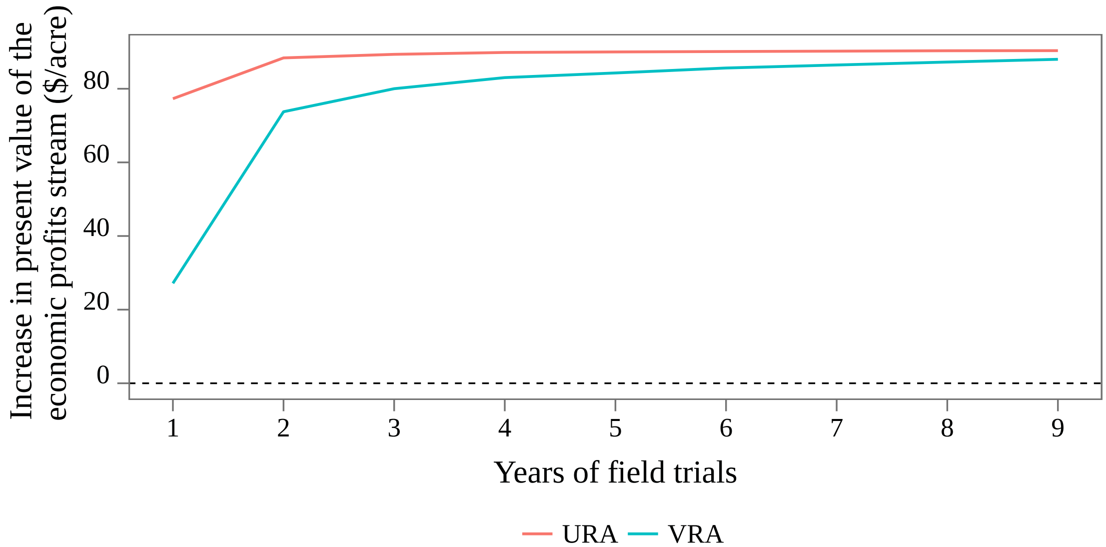

Figure 9: Increase in the present value of profits stream due to information provision (v_L, y_L)

Note: Mathematically, URA and VRA represent $E[\Pi^{info}(v_L, y_L, URA)]$ and $E[\Pi^{info}(v_L, y_L, VRA)]$.

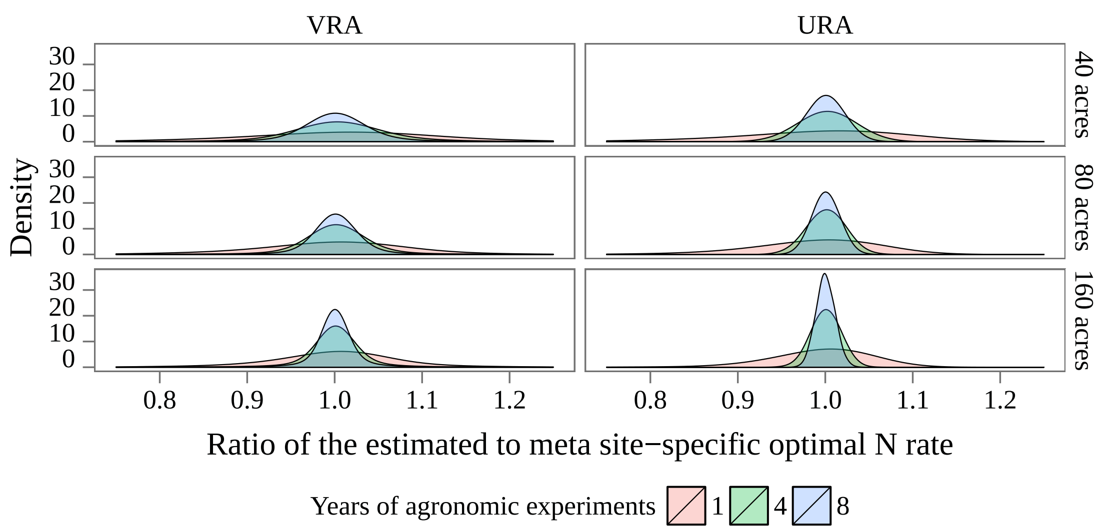

Figure 10: Convergence of estimated optimal N rates (VRA and URA) to the meta optimal N rates by field size

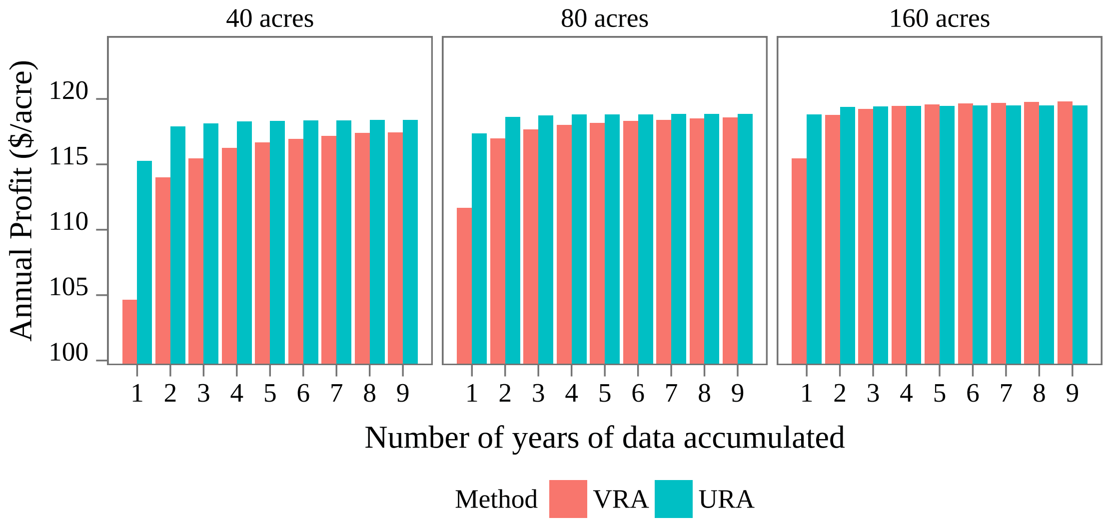

Figure 11: Annual profitability of VRA and URA by field size

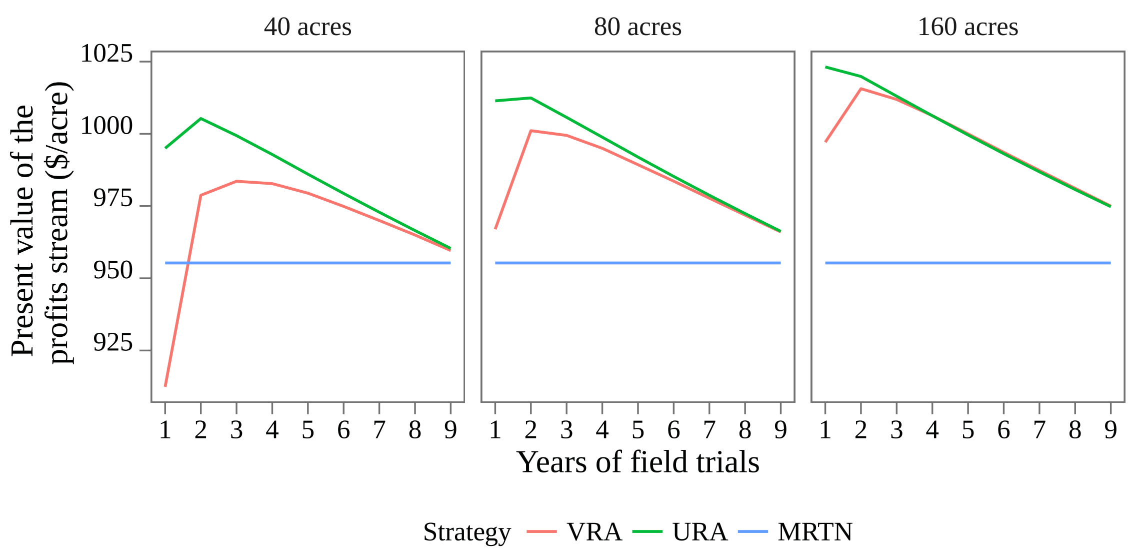

Figure 12: Present value of profit streams under VRA, URA, and MRTN by field size

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