# Oligopoly Intermediation, Relative Rivalry, and the Mode of Competition

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August 13, 2013

**Abstract**

Policy design in oligopolistic settings depends critically on the mode of competition between firms. We develop a model of oligopoly intermediation that reveals the mode of competition to be an equilibrium outcome that depends on the relative degree of rivalry between firms in the upstream and downstream markets. We examine two forms of sequential pricing games: Purchasing to stock (PTS), in which firms select input prices prior to setting consumer prices; and purchasing to order (PTO), in which firms sell forward contracts to consumers prior to selecting input prices. The equilibrium outcomes of the model range between Bertrand and Cournot depending on the relative degree of rivarly between firms in the upstream and downstream markets. Prices are strategic complements and the equilibrium prices coincide with the Bertrand outcome when the markets are equally rivalrous, while prices are strategic substitutes when the degree of rivalry is sufficiently high in one market relative to the other. Cournot outcomes emerge under circumstances in which prices are strategically independent in either the upstream or downstream market. We derive testable implications for the mode of competition that depend only on primitive conditions of supply and demand functions.

*JEL Classification*: L13, L22, F13
*Keywords*: Oligopoly; Intermediation; Strategic Pre-commitment Policy.

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# 1 Introduction

An obstacle to deriving policy implications from the oligopoly model is the sensitivity of strategic pre-commitment devices to the mode of competition. As observed by Fudenberg and Tirole (1984) and Bulow, Geanakoplos, and Klemperer (1985), the strategic underpinnings of the oligopoly model depend fundamentally on the manner in which firms' choice variables alter the marginal profit expressions of rivals, a feature that has essential implications in a number of policy settings. For strategic trade policy, export subsidies are optimal when firms' choice variables are strategic substitutes (Brander and Spencer 1985), but export taxes are optimal when firms' choice variables are strategic complements (Eaton and Grossman 1986). For strategic delegation, the optimal managerial contract overcompensates sales when agents compete in strategic substitutes, but overemphasizes profits when agents compete in strategic complements (Fershtman and Judd 1987, Sklivas 1987). For contracts between vertically aligned suppliers, the optimal contract involves lump sum transfers from manufacturers to retailers when retailers compete in strategic complements (Shaffer 1991), but involves negative lump sum payments when retailers compete in strategic substitutes (Vickers 1985).[1]

In this paper, we construct an oligopoly framework that generates testable hypotheses on the mode of competition. We frame our model around duopoly intermediaries who purchase an input from a common upstream market and sell finished goods in a common downstream market. The intermediated oligopoly model provides a convenient way to represent strategic interaction between firms: It reduces to the usual oligopoly (oligopsony) model when the supply (demand) function is infinitely elastic, yet encompasses general forms of strategic interaction when prices are interdependent in both supply and demand functions.

We characterize the degree of rivalry between firms by allowing products to be

[1] The mode of competition of the oligopoly model also has important implications for first-mover advantage in sequential games between firms. It has been known since von Stackelberg (1934) that advantage goes to the first-mover when choice variables are strategic substitutes; however, being the first-mover is disadvantageous in the case of strategic complements (Gal-Or 1985).

differentiated in the upstream and downstream markets, as would be the case when manufacturers rely on specialized inputs to produce branded consumer goods. Our setting thus extends the analysis of Stahl (1988) to differentiated product markets. As in Stahl's intermediation model, we consider two forms of sequential price competition: (*i*) "purchasing to stock" (PTS), in which the firms select input prices prior to setting output prices; (*ii*) "purchasing to order" (PTO), in which the firms sell forward contracts to consumers prior to selecting input prices. We compare outcomes from these sequential-pricing models to the Bertrand ("Nash-in-margins") outcome in which the firms simultaneously select upstream and downstream market prices.

Our observations on the mode of competition depend critically on what we refer to as the "relative degree of market rivalry" in the upstream and downstream markets. We define the relative rivalry of the upstream market to be the difference (in absolute terms) between the ratio of the cross-price to own-price elasticity of supply and the ratio of cross-price to own-price elasticity of demand. It is a measure of the relative strength of the oligopoly interaction at each point of contact between firms. When the cross-price elasticity is zero (the monopoly case), there is no strategic interaction between firms in the market, and when the ratio of cross-price elasticity to own-price elasticity approaches one, the market is highly rivalrous. Markets are equally rivalrous when there is no difference in the intensity of the oligopoly interaction in the upstream and downstream markets, as would be the case in the setting considered by Stahl (1988) with homogeneous products in each market, while the upstream market is relatively rival in cases such as when a homogeneous input is used to produce differentiated consumer goods.

Our main results can be summarized as follows. First, we find that the Bertrand equilibrium emerges only under circumstances in which the upstream and downstream markets are equally rivalrous. This outcome occurs irrespective of the degree of product differentiation in each market. Thus, our analysis reveals the underpinning of the Stahl (1988) outcome of Bertrand merchants to be determined by the *relative* degree of market rivalry rather than the *absolute* degree of rivalry in the upstream and downstream markets.

Second, we show firm profits to be greater under PTS than under Bertrand or PTO when the downstream market is relatively rivalrous, whereas the PTO outcome Pareto dominates Bertrand and PTS outcomes when the upstream market is relatively rivalrous. In either case, setting prices in the less rivalrous market serves as a pre-commitment device for prices in the more rivalrous market, providing firms with the ability to relax price competition in the market where the degree of oligopoly interaction is most intense.

Third, we demonstrate that when one market is more rivalrous than the other, prices can be either strategic complements or strategic substitutes depending on the relative degree of rivalry. If the degree of rivalry relaxes one of the markets from an initial Bertrand position with equal rivalry, the equilibrium outcome converges to Cournot as prices become strategically independent in the less rivalrous market. This outcome is related to the finding of Kreps and Scheinkman (1983) in a two-stage game where firms first choose capacity and then select prices, and we show their essential observation to hold irrespective of whether prices are strategically independent in the upstream market or the downstream market.[2]

Fourth, we provide conditions under which our observations on oligopoly intermediation are robust to inventory-holding behavior in symmetric markets with linear supply and demand functions. Our analysis thus extends the work of Kreps and Scheinkman (1983) to markets with differentiated products by building on the analysis of Martin (1999).

Finally, we expand our analysis of sequential pricing outcomes to an extended game in which firms first choose the timing of pricing decisions prior to selecting prices. In the case of symmetric markets with linear supply and demand functions, we verify that the Pareto dominant outcomes in the pricing sub-game represent equi-

[2] In general, the role of input prices as a pre-commitment device to soften downstream price competition differs from the quantity choice problem considered by Kreps and Scheinkman (1983). In the intermediation model, oligopoly firms face upward-sloping supply functions and input prices are strategically inter-dependent, which results in a continuum of equilibrium outcomes.

librium strategies in the extended game. Under circumstances in which firm profits are largest under PTS (PTO), such a sequential pricing strategy is also the equilibrium outcome of the extended game.

Our observations are related to previous research by Maggi (1996) that has endogenized the mode of competition in the oligopoly model. In Maggi's model, firms make capacity commitments before trading in a downstream international market, but can subsequently relax their prior capacity commitments subject to an *ex post* adjustment cost parameter. This sequential capacity adjustment process produces a continuum of outcomes that spans the modes of competition between Bertrand and Cournot according to the cost of capacity adjustment. Our framework departs from Maggi (1996) by specifying oligopsony interaction in the input (capacity) market in place of *ex post* adjustment cost. An advantage of this approach is that our model results in testable hypotheses on the mode of competition at the industry level that are readily estimable from market data.

We illustrate the policy implications of the model for the case of a contract between a principle (a domestic trade authority, a firm, or a controlling shareholder) and an agent (a domestic firm, a supplier/consumer, or a manager) that imposes a unit tax or subsidy on the input procured from the upstream market. We show that the optimal contract to maximize oligopoly profits involves taxing the input when the upstream and downstream markets are relatively equal in terms of rivalry, but subsidizing the input when the degree of relative rivalry is sufficiently large. We numerically characterize these policy outcomes for perturbations in the relative degree of market rivalry under linear supply and demand conditions and show that the optimal value of the policy variable follows an inverted $u$-shaped pattern: As the upstream market becomes increasingly rivalrous, the optimal policy switches from a subsidy to a tax under PTS before reverting back to a subsidy under PTO.

The remainder of the paper is structured as follows. In the next Section we present the model and characterize equilibrium prices under Bertrand, PTS and PTO. In Section 3, we compare the outcomes for firm and industry profits and classify the Pareto dominant Nash equilibrium according to the relative degree of rivalry in the upstream and downstream markets. In Section 4, we extend these outcomes to consider inventory-holding behavior and the choice of sequential or simultaneous price selection in a symmetric model with linear supply and demand conditions. In Section 5, we characterize the mode of competition in the linear model and provide a condition for prices to be strategic substitutes that depends only on observed market prices and estimable supply and demand elasticities in a particular industry. In Section 6, we numerically illustrate the implication of our findings for the case of vertical contracts between firms and their upstream suppliers, and in Section 7, we conclude. The proofs of all Propositions appear in the Appendix.

## 2 The Model

We consider duopoly intermediaries, labeled firms 1 and 2, who compete against each other in prices. The firms purchase differentiated stocks ("inputs") from suppliers in an upstream market, and sell finished products ("outputs") derived from the inputs to consumers in a downstream market. Both input suppliers and consumers are price-taking agents in their respective markets.

To clarify the implications of the model for oligopoly pricing outcomes, we consider fixed proportions technology. Specifically, letting $x_i$ denote the quantity of the input purchased in the upstream market by firm $i$, we scale units such that $y_i = x_i$ denotes the quantity of the output sold by the firm in the downstream market. Products in each market are differentiated, and the degree of rivalry between firms potentially differs at their points of contact in the upstream and downstream markets according to the intensity of cross-price effects between firms in each market. One interpretation of the model is that demand and supply functions depend on the spatial location of suppliers, consumers, and firms. Another interpretation is that the firms require specialized inputs to produce differentiated consumer goods.

Let $\mathbf{p} = (p_1, p_2)$ denote the vector of output prices. Consumer demand for product

$i$ is given by

$$D^i = D^i(\mathbf{p}), \qquad i = 1, 2, \tag{1}$$

where $D^i$ is twice continuously differentiable. We assume $D^i_i \equiv \partial D^i/\partial p_i < 0$ and $D^i_j \equiv \partial D^i/\partial p_j \geq 0$, where the latter condition confines attention to the case of substitute goods.

Let $\mathbf{w} = (w_1, w_2)$ denote the vector of input prices, so that the supply function facing firm $i$ in the upstream market is

$$S^i = S^i(\mathbf{w}), \qquad i = 1, 2, \tag{2}$$

where $S^i$ is twice continuously differentiable and $S^i_i \equiv \partial S^i/\partial w_i > 0$ and $S^i_j \equiv \partial S^i/\partial w_j \leq 0$ (i.e., products in the upstream market are substitutes).

Throughout the paper, we assume that the direct effect of a price change outweighs the indirect effect in each market; that is, $\Delta \equiv D^i_i D^j_j - D^i_j D^j_i > 0$ and $\Sigma \equiv S^i_i S^j_j - S^i_j S^j_i > 0$. These conditions ensure that the system of demand and supply equations is invertible. We also assume sufficient conditions are met for existence and uniqueness of the pure strategy Nash equilibrium under PTS and PTO. These conditions always hold under the restrictions above for the case of linear supply and demand. For the general case, we impose the (mild) regularity conditions on profits in the PTS game, $\pi^{i,s}(w_i, w_j)$, that $\lim_{w_i \to 0} \pi^{i,s}_i(w_i, w_j) > 0$, $\lim_{w_i \to +\infty} \pi^{i,s}_i(w_i, w_j) < 0$, and $\pi^{i,s}_{ii}(w_i, w_j) + \pi^{i,s}_{ij}(w_i, w_j) < 0$ , and on profits in the PTO game, $\pi^{i,o}(p_i, p_j)$, that $\lim_{p_i \to 0} \pi^{i,o}_i(p_i, p_j) > 0$, $\lim_{p_i \to +\infty} \pi^{i,o}_i(p_i, p_j) < 0$, and $\pi^{i,o}_{ii}(p_i, p_j) + \pi^{i,o}_{ij}(p_i, p_j) < 0$. In Section 4.1, we provide sufficient conditions for existence and uniqueness of the PTS equilibrium in the linear case with inventory-holding behavior.

The concept of relative rivalry is important for the analysis to follow. We define the relative rivalry of markets as a measure of the strength of the oligopoly interaction in the upstream market relative to the downstream market, where a more rivalrous market is one in which a change in the input (output) price selected by a firm leads to a greater change in supply (demand) for the rival. Focusing on the symmetric market equilibrium, we describe the relative rivalry of markets in terms of supply and demand elasticities. Specifically, let $\epsilon_s = \frac{\varepsilon_c^s}{\varepsilon_o^s} < 1$ denote the absolute value of the ratio of cross-price elasticity of supply ($\varepsilon_c^s = -S_j^i \frac{w_j}{S^i} > 0$) to own-price elasticity of supply ($\varepsilon_o^s = S_i^i \frac{w_i}{S^i} > 0$), and let $\epsilon_d = \frac{\varepsilon_c^d}{\varepsilon_o^d} < 1$ denote the corresponding ratio of demand elasticities in the downstream market, where $\varepsilon_c^d = D_j^i \frac{p_i}{D^i} > 0$ and $\varepsilon_o^d = -D_i^i \frac{p_i}{D^i} > 0$.

**Definition 1** *The relative rivalry of the upstream market is given by* $\Theta = \epsilon_s - \epsilon_d$.

We measure the relative rivalry of the upstream market according to equilibrium values for the prices described below. We refer to the upstream market as being more rivalrous than the downstream market when $\Theta > 0$, the downstream market as being more rivalrous than the upstream market when $\Theta < 0$, and the markets as being equally rivalrous when $\Theta = 0$.

In Section 4.2, we derive conditions for strategic inventory-holding behavior. For now, we streamline the exposition of equilibrium outcomes under PTS and PTO by suppressing inventory-holding behavior and the destruction or removal of goods. Without the possibility of holding inventory, the demand and supply functions facing each firm are linked by the material balance equation,

$$D^i(\mathbf{p}) = S^i(\mathbf{w}), \qquad i = 1, 2. \tag{3}$$

We are now ready to examine the equilibrium outcomes of the intermediation model under two forms of sequential pricing behavior: (*i*) Purchasing to Stock (PTS); and (*ii*) Purchasing to Order (PTO). As a convenient benchmark for this analysis, we first examine the Nash in margins equilibrium in which firms simultaneously select prices in the upstream and downstream markets. Throughout the paper, we refer to the Nash in margins equilibrium as the Bertrand outcome.

## 2.1 Bertrand Outcomes

Consider the case in which firms select prices simultaneously in the upstream and downstream markets. Firm $i$ seeks to maximize profits

$$\pi^{i,B}(\mathbf{p}, \mathbf{w}) = p_i D^i(\mathbf{p}) - w_i S^i(\mathbf{w}) - F_i, \qquad i = 1, 2 \tag{4}$$

subject to the inventory constraint (3). Evaluating the first-order conditions with respect to $w_i$ and $p_i$, the Bertrand equilibrium satisfies

$$p_i - w_i = \frac{S^i(\mathbf{w})}{S^i_i(\mathbf{w})} - \frac{D^i(\mathbf{p})}{D^i_i(\mathbf{p})}, \qquad i = 1, 2, \tag{5}$$

where subscripts refer to partial derivatives.

Simultaneously solving (5) subject to the inventory constraint (3) yields the Bertrand equilibrium prices, which we define in the symmetric case as $(w^B, p^B)$. For future reference, it is helpful to express the symmetric equilibrium price-cost margin under Bertrand as

$$p^B - w^B = \frac{p^B}{\varepsilon^d_o} + \frac{w^B}{\varepsilon^s_o}. \tag{6}$$

## 2.2 Purchasing to Stock (PTS)

In the purchasing to stock (PTS) game, firms first select input prices and acquire stocks in the upstream market, and then subsequently select output prices for finished goods in the downstream market. Let $\mathbf{w} = (w_1, w_2)$ denote the vector of input prices selected by the firms and let $p^i(\mathbf{w})$ and $p^j(\mathbf{w})$ denote the associated output prices implicitly defined by equations (3).

Under PTS, firm $i$ selects $w_i$ to maximize profits,

$$\pi^{i,s}(w_i, w_j) = \left(p^i(\mathbf{w}) - w_i\right) S^i(\mathbf{w}) - F_i, \qquad i = 1, 2, \tag{7}$$

where $F_i$ denotes fixed costs, a portion of which may be sunk. The first-order necessary condition for a profit maximum is

$$\pi^{i,s}_i \equiv (p^i(\mathbf{w}) - w_i)S^i_i(\mathbf{w}) + (p^i_i(\mathbf{w}) - 1)S^i(\mathbf{w}) = 0, \qquad i = 1, 2. \tag{8}$$

The effect of an input price change by firm $i$ on output prices can be derived by totally differentiating equations (3). Holding $dw_j = 0$, this yields

$$\begin{bmatrix} D^i_i & D^i_j \\ D^j_i & D^j_j \end{bmatrix} \begin{bmatrix} dp_i \\ dp_j \end{bmatrix} = \begin{bmatrix} S^i_i \\ S^j_i \end{bmatrix} dw_i.$$

It follows that

$$p^i_i(\mathbf{w}) \equiv \frac{\partial p^i}{\partial w^i} = \frac{D^j_j S^i_i - D^i_j S^j_i}{\Delta} < 0$$

$$p_i^j(\mathbf{w}) \equiv \frac{\partial p^j}{\partial w^i} = \frac{D_i^i S_i^j - D_i^j S_i^i}{\Delta}. \tag{9}$$

Equation (2.2) represents the own-price effect of an increase in the input price on the output price of firm $i$. This term is negative. When firm $i$ raises his input price, the firm procures a greater quantity of the input, and this drives down the firm's output price and narrows his price-cost margin.

Equation (9) is the cross-price effect of a input price increase by firm $i$ on the output price of the rival firm $j$. The sign of this term is important for the results to follow. Firm $j$ responds to a higher input price by firm $i$ ($dw_i > 0$) by increasing his output price ($dp_j > 0$) whenever $D_i^i S_i^j > D_i^j S_i^i$, and otherwise holds constant or decreases his output price.

Under PTS, a change in a firm's input price has two offsetting effects on the output price of the rival. The first effect is the "supply effect", $D_i^i S_i^j dw_i \geq 0$. Selecting a higher input price bids stocks away from the rival in the upstream input market, which leads to an inward shift of the rival's supply function. The supply effect reduces the rival's procurement of the input, thereby reducing the quantity sold in the downstream market by firm $j$ and raising the rival's output price. The second effect is the "demand effect", $D_i^j S_i^i dw_i \geq 0$. A rise in the input price of firm $i$ raises input procurement for firm $i$ in the upstream market, leading to a commensurate increase in production and sales for firm $i$ in the downstream market and a decrease in $p_i$. The demand effect results in an inward shift of the rival's demand function, placing downward pressure on $p_j$. The relative magnitude of the supply and demand effects depends on the relative rivalry of the upstream market, for instance the demand effect dominates the supply effect when products in the downstream market are homogeneous as a unilateral increase in the input price of one firm raises total production, flooding the downstream market with finished goods.

In the symmetric market equilibrium ($D^i = D^j, S^i = S^j, p_1 = p_2 = p, w_1 = w_2 = w$), this condition can be expressed in terms of relative rivalry as

$$p_i^j(\mathbf{w}) \stackrel{s}{=} \Theta,$$

where " $\stackrel{s}{=}$ " denotes"equal in sign". Under circumstances in which the upstream market is relatively rivalrous ($\Theta > 0$), an increase in the input price by firm $i$ increases the output price of firm $j$. When the downstream market is relatively rivalrous ($\Theta < 0$), an increase in the input price by firm $i$ decreases the output price of firm $j$, and for an equal degree of rivalry, $\Theta = 0$, the output price of firm $j$ is independent of the input price selection of firm $i$.

Relative rivalry has essential strategic implications for the oligopoly model. To see this, note that

$$S^i(w_i, w_j) = D^i(p_i(\mathbf{w}), p_j(\mathbf{w}))$$

under the inventory constraint (3). Differentiating this expression with respect to $w_i$ and dropping arguments for notational convenience, we have

$$S_i^i = D_i^i p_i^i + D_j^i p_i^j.$$

When firm $i$ increases his input price in the upstream market, this leads to a direct increase in the output sold by firm $i$, $D_i^i p_i^i > 0$. But the input price increase also facilitates a strategic response by the rival firm, $D_j^i p_i^j \lesseqgtr 0$, the sign of which depends on $p_i^j$ (and thus on $\Theta$). Specifically, firm $i$ perceives a smaller supply response to a rise in $w_i$ when $\Theta < 0$, which reduces the value of increasing his input price. Thus, engaging in PTS behavior serves as a commitment device to refrain from increasing input prices when $\Theta < 0$. It is of course in the interest of both firms to maintain lower input prices in the upstream market, as this supports correspondingly higher output prices in the downstream market and higher price-cost margins, making sequential pricing behavior under PTS a facilitating practice whenever $\Theta < 0$.

The equilibrium under PTS is determined by the simultaneous solution of equations (8). Let $\mathbf{w}^s = (w_1^s, w_2^s)$ denote the equilibrium input price vector that solves these equations and let $\mathbf{p}^s = (p_1^s, p_2^s)$ denote the associated vector of equilibrium output prices implied by equations (3).

The equilibrium price-cost margin for symmetric firms can be written

$$p^s - w^s = k^s \frac{p^s}{\varepsilon_o^d} + \frac{w^s}{\varepsilon_o^s}, \tag{10}$$

where $k^s = 1+\frac{D_i^j D_j^j \Theta}{\Delta}$. Notice that the second term on the right-hand side of equation (10) is identical to the second term on the right hand-side of equation (6), but that the first term differs from the Bertrand equilibrium margin. Playing PTS introduces a weight on the "demand-side" portion of the equilibrium price-cost margin that jointly accounts for the relative rivalry of the upstream market. By inspection, $k^s > 1$ if and only if $\Theta < 0$. That is, equilibrium price-cost margins are higher for firms in the PTS game than in the Bertrand outcome when the upstream market is less rivalrous than the downstream market ($\Theta < 0$). The intuition for this outcome is quite clear: When $\Theta < 0$, a decrease in the input price by a firm facilitates an increase in the rival's output price, thereby softening price competition in the downstream market.

In Section 3, we provide a more complete comparison of PTS and Bertrand outcomes. Before turning to this analysis, we derive the market equilibrium for the remaining case of PTO.

### 2.3 Purchasing to Order (PTO)

Suppose the firms sell forward contracts for delivery of finished goods to consumers prior to procuring inputs in the upstream market. Forward contracts are widely used in practice, including imported goods sold to retail distributors and a significant portion of wholesale trade (Stahl 1988).

Let $\mathbf{p} = (p_1, p_2)$ denote the vector of output prices selected by the firms and let $w^i(\mathbf{p})$ and $w^j(\mathbf{p})$ denote the associated input prices defined by equations (3). In the PTO game, firm $i$ selects his output price to maximize profits of

$$\pi^{i,o}(p_i, p_j) = \big(p_i - w^i(\mathbf{p})\big)\, D^i(\mathbf{p}) - F_i, \qquad i = 1, 2.$$

The first-order necessary condition for a profit maximum is

$$\pi_i^{i,o} \equiv (p_i - w^i(\mathbf{p}))D_i^i(\mathbf{p}) + (1 - w_i^i(\mathbf{p}))D^i(\mathbf{p}) = 0, \qquad i = 1, 2. \tag{11}$$

We evaluate the PTO equilibrium by proceeding as above. Making use of the implicit function theorem on equations (3) gives the input price responses

$$w_i^i(\mathbf{p}) \equiv \frac{\partial w^i}{\partial p^i} = \frac{D_i^i S_j^j - D_i^j S_j^i}{\Sigma} < 0 \tag{12}$$

$$w_i^j(\mathbf{p}) \equiv \frac{\partial w^j}{\partial p^i} = \frac{D_i^j S_i^i - D_i^i S_i^j}{\Sigma}. \tag{13}$$

Equation (12) measures the own-price effect of an increase in the output price, and equation (13) measures the cross-price effect of a output price change by firm $i$ on the input price selected by firm $j$. The interpretation of the latter cross-price effect is synonymous with the interpretation under PTS and the sign of this terms depends on the relative magnitude of the supply effect, $D_i^i S_i^j dp_i \geq 0$, and the demand effect, $D_i^j S_i^i dp_i \geq 0$.

Notice that the cross-price effect in equation (13) always takes the opposite sign of the cross-price effect in equation (9); that is, $p_i^j(\mathbf{w}) \stackrel{s}{=} -w_i^j(\mathbf{p})$. Under conditions in which an increase in a firm's input price increases the output price of his rival in the PTS game, an increase in the output price decreases the input price of his rival in the PTO game. In the symmetric market equilibrium $(D^i = D^j, S^i = S^j, p_1 = p_2 = p, w_1 = w_2 = w)$,

$$w_i^j(\mathbf{p}) \stackrel{s}{=} -\Theta.$$

The equilibrium under PTO is determined by the simultaneous solution of equations (11). Define the equilibrium output price vector that solves these equations as $\mathbf{p}^o = (p_1^o, p_2^o)$ and the associated vector of equilibrium input prices as $\mathbf{w}^o = (w_1^o, w_2^o)$.

The equilibrium price-cost margin for symmetric firms can be written

$$p^o - w^o = \frac{p^o}{\varepsilon_o^d} + k^o \frac{w^o}{\varepsilon_o^s}.$$

where $k^o = 1 - \frac{S_i^j S_j^j}{\Sigma}\Theta$. Inspection of this term reveals that $k^o > 1$ if and only if $\Theta > 0$. Equilibrium price-cost margins are higher in the PTO game than in the Bertrand outcome when the upstream market is relatively rivalrous $(\Theta > 0)$. The reason is that a rise in the output price by a firm decreases the input price set by his rival when $\Theta > 0$, which facilitates higher price-cost margins.

# 3 Equilibrium Outcomes

In this Section we compare firm profits in the PTS and PTO games to the Bertrand outcome and identify Pareto dominant strategies in the symmetric market equilibrium. To do so, we consider the symmetric Bertrand prices $(w^B, p^B)$ and examine multilateral defections from the equilibrium that increase the profits of firms.[3]

## 3.1 PTS Versus PTO

Consider first the PTS game. Evaluating the input price condition (8) at the symmetric Bertrand equilibrium position $(w^B, p^B)$ gives

$$\pi_i^{i,s}(w^B, p^B) \equiv \left( \frac{D_j^j S_i^i - D_j^i S_i^j}{\Delta} - 1 \right) S^i - D^i \left( \frac{S_i^i}{D_i^i} \right),$$

where all terms are evaluated at $(w^B, p^B)$. Making use of the market-clearing condition (3) and factoring terms yields

$$\pi_i^{i,s}(w^B, p^B) \equiv \frac{S^i D_j^i S_i^i}{\Delta} \Theta \overset{s}{=} \Theta.$$

In the PTS game, firms select higher input prices than in the symmetric Bertrand equilibrium when $\Theta > 0$ and select lower input prices when $\Theta < 0$.

Proceeding similarly in the case of PTO, evaluating the output price condition (11) at the symmetric Bertrand equilibrium position $(w^B, p^B)$ gives

$$\pi_i^{i,o}(w^B, p^B) \equiv \frac{D^i D_i^i S_j^i}{\Sigma} \Theta \overset{s}{=} \Theta.$$

In the PTO game, firms select higher output prices than in the symmetric Bertrand equilibrium when $\Theta > 0$ and set lower output prices when $\Theta < 0$.

**Proposition 1** *For the intermediated oligopoly model:*

**(i)** *If $\Theta = 0$ the equilibrium market outcome under either PTO or PTS is Bertrand*

**(ii)** *If $\Theta > 0$ $(< 0)$ the Pareto dominant equilibrium is PTO (PTS).*

[3] With slight abuse of notation, we write demand, supply, and profit as functions of the scalar values of input and output prices in the symmetric equilibrium.

In Stahl's (1988) model, intermediaries compete in homogeneous product markets ($\Theta = 0$), and an outcome with Bertrand merchants emerges under both PTO and PTS. Proposition 1 extends this outcome to encompass any market that satisfies $\Theta = 0$. Thus, the Bertrand equilibrium represents an envelope of oligopoly outcomes characterized by equal degrees of rivalry in the upstream market and downstream market. The essential underpinning of the Bertrand outcome is that firms' input (output) price choices are independent of the resulting output (input) prices selected by the rival. This outcome depends on the relative degree (rather than the absolute degree) of market rivalry.

When $\Theta \neq 0$, firm profits are larger in the symmetric equilibrium in cases where the firms enjoy wider price-cost margins. Under circumstances where the downstream market is relatively rivalrous ($\Theta < 0$), the PTS game facilitates this outcome, because the rival responds to a lower input price by selecting a higher output price in equation (9). Equilibrium price-cost margins and profits are accordingly higher under PTS than under Bertrand. Under circumstances where the upstream market is relatively rivalrous ($\Theta > 0$), profits are higher in the PTO game, as the rival firm responds to a higher output price in this case by selecting a lower input price in equation (13). In either case, the Pareto dominant equilibrium involves selecting prices in the relatively less rivalrous market as a facilitating practice to soften price competition in the remaining market.

The anatomy of the intermediated oligopoly model can be illustrated by describing circumstances in which playing the PTS (PTO) game produces Cournot outcomes. In the following section we characterize the Cournot equilibrium and derive formal conditions under which the PTS and PTO games produce Cournot outcomes.

### 3.2 Cournot Outcomes

Let $\mathbf{y} = (y_1, y_2)$ denote the vector of retail (and wholesale) quantities under quantity competition. Defining inverse demand and inverse supply for product $i$ as $P^i(\mathbf{y})$ and

$W^i(\mathbf{y})$, respectively, the profit of firm $i$ under Cournot competition is given by

$$\pi^{i,C}(y_i, y_j, F_i) = \left(P^i(\mathbf{y}) - W^i(\mathbf{y})\right) y_i - F_i, \qquad i = 1, 2.$$

The first-order necessary condition for a profit maximum is

$$\pi_i^{i,C}(y_i, y_j, F_i) = P^i - W^i + \left(\frac{\partial P^i}{\partial y_i} - \frac{\partial W^i}{\partial y_i}\right) y_i = 0, \qquad i = 1, 2. \tag{14}$$

Simultaneously solving equations (14) in the symmetric equilibrium gives the equilibrium quantity, $y^C = y_1^C = y_2^C$, which can be used to recover the symmetric Cournot equilibrium prices $(w^C, p^C)$.

**Proposition 2** *The Cournot equilibrium emerges in the PTS game when $\epsilon_s = 0$ and in the PTO game when $\epsilon_d = 0$.*

The PTS (PTO) game produces Cournot oligopoly (oligopsony) outcomes in cases where prices in the upstream (downstream) market are strategically independent. Proposition 2 clarifies the essential finding of Kreps and Scheinkman (1983) as an outcome that depends on the strategic independence of prices in the upstream input (capacity) market in the PTS game. The Cournot outcome also emerges in the PTO game when the output prices of firms are strategically independent. In the following section, we illustrate the robustness of these outcomes by considering circumstances in which firms can hold inventories.

To see the intuition for Proposition 2, consider the price-cost margins under Cournot in the symmetric market equilibrium. Making use of equations (2.2) and (12), the symmetric equilibrium price-cost margin can be written as

$$p^C - w^C = \frac{p^C}{\varepsilon_o^d\left(1 - \epsilon_d^2\right)} + \frac{w^C}{\varepsilon_o^s\left(1 - \epsilon_s^2\right)}. \tag{15}$$

The essential difference between this outcome and the outcome under PTS and PTO is that quantity-setting firms jointly consider the strategic effect of a quantity increase on raising their procurement cost in the upstream market and on reducing their sales revenue in the downstream market. In contrast, firms in the PTS equilibrium, who set prices sequentially in the upstream market prior to selecting prices in the downstream market, can consider only the implication of their first-stage input price choice on their subsequent level of sales. When prices in the upstream market are strategically independent, $\epsilon_s = 0$, however; the only interaction that remains with the rival in this case occurs through the interdependence of prices in the downstream oligopoly market, so that the inability of the firm to account for the strategic interdependence of input prices in the upstream market no longer has any consequence. When $\epsilon_s = 0$, the second term on the right-hand side of equation (15) reduces to $\frac{w^C}{\varepsilon_o^s}$ and the weight on the demand-side portion of the equilibrium price-cost margin in the PTS game reduces to $k^s = \frac{1}{1-\epsilon_d^2}$, resulting in the Cournot outcome. For a similar reason, the PTO game results in the Cournot oligopsony outcome when prices are strategically independent in the downstream market, $\epsilon_d = 0$.

# 4 Model Extensions

Thus far, our analysis considers only the Pareto dominance of profit outcomes in settings that preclude the possibility of holding inventory. In this Section, we extend the model to consider inventory-holding behavior.

Before turning to the possibility that firms hold positive inventories, we first extend the game structure to a setting in which firms endogenously select the timing of their pricing decisions. Specifically, we consider a game in which firms first choose among the alternatives of simultaneous price selection (Bertrand) and sequential price selection (PTS or PTO) prior to choosing prices.

For each extension, we examine the symmetric market equilibrium under conditions of linear supply and demand. Specifically, we consider the linear specialization,

$$D^i(\mathbf{p}) = \max\{a - bp_i + cp_j, 0\}, \tag{16}$$

$$S^i(\mathbf{w}) = \max\{\beta w_i - \gamma w_j, 0\}, \tag{17}$$

where $i = 1, 2, i \neq j$, and where $a, b, \beta > 0$ and $c, \gamma \geq 0$ are positive constants. We restrict the each firm's prices to the interval $[0, P]$, where $P \geq 2a/(b - c)$. Demand (supply) conditions reduce to local monopoly (monopsony) markets when $c = 0$ (

$\gamma = 0$) and products in the downstream (upstream) market become increasingly commoditized as $c \longrightarrow b$ ($\gamma \longrightarrow \beta$).

### 4.1 Endogenous Timing

Our observations above on the profit motive of firms to select prices sequentially in intermediated oligopoly markets raises the question of whether such behavior is also an equilibrium strategy in settings where coordination on timing is not possible prior to selecting prices. Here, we extend the game structure to a setting in which firms first choose whether to engage in simultaneous price selection (Bertrand) or set prices sequentially (PTS or PTO) prior to selecting prices in each market.

Consider the following three stage game. In stage 1, firms select the timing of their pricing decisions from the choice set $\{Bertrand, PTS, PTO\}$. In stage 2, firms select prices in the relevant subgame according to the timing of pricing decisions determined in stage 1. The firms face symmetric market conditions in the second stage according to the demand and supply functions in equations (16) and (17).

Our comparison of stage 2 outcomes involves examining equilibrium prices under several possible alternatives: ($i$) firm $i$ plays PTS while firm $j$ plays PTO; ($ii$) firm $i$ plays Bertrand while firm $j$ plays PTS; ($iii$) firm $i$ plays Bertrand while firm $j$ plays PTO; ($iv$) both firms play Bertrand; ($v$) both firms play PTS; and ($vi$) both firms play PTO. We have completely characterized the equilibrium outcomes of the latter three subgames and we turn here to cases in which firms select asychronous prices. In cases where firm $i$ plays Bertrand and firm $j$ plays PTS or PTO, we consider a timing of pricing behavior that involves firm $i$ simultaneously selecting the prices $(w_i, p_i)$, while firm $j$ sets either $w_j$ (PTS) or $p_j$ (PTO) prior to selecting the remaining price in the pricing subgame according to the response function $p^j(w_i, p_i, w_j)$ under PTS or $w^j(w_i, p_i, p_j)$ under PTO. To motivate this setting for asychronous prices, it is helpful to consider the choice of sequential timing (PTS or PTO) as a defection strategy from the Bertrand outcome. To implement a defection strategy from the Bertrand equilibrium, firm $j$ delays the selection of his output price (PTS) or input

| **Firm $i$'s strategy** | **Bertrand** | **PTS** | **PTO** |
|---|---|---|---|
| Firm $j$'s margin $(p_j - w_j)$ | $\frac{S^j}{S^j_j} - \frac{D^j}{D^j_j}$ | $\frac{S^j}{S^j_j} - \frac{D^j}{D^j_j} k^s$ | $\frac{S^j}{S^j_j} k^o - \frac{D^j}{D^j_j}$ |

Table 1: Price-Cost Margins for Firm j in Response to the Timing of Firm i

price (PTO) in the pricing subgame by selecting only one price in response to the simultaneous prices selected by the rival.

Our analysis of cases $(i)$-$(iii)$ yields the following result, which holds under general supply and demand conditions.

**Proposition 3** *The selection of a given timing strategy $\{Bertrand, PTS, PTO\}$ by firm $i$ provides the rival firm $j$ with an equilibrium price-cost margin that corresponds with the strategy selected by firm $i$.*

Table 1 describes the different possibilities for the equilibrium price cost margin for firm $j$ depending on the firm $i$'s strategy. The entries in Table 1 have important strategic implications. Recall that $k^o = 1 - \frac{S^i_j S^j_j}{\Sigma}\Theta$ and $k^s = 1 + \frac{D^j_i D^j_j}{\Delta}\Theta$. If firm $i$ defects from the Bertrand equilibrium to play PTS, the defection has no implications for firm $i$'s price-cost margin, which remains at the Bertrand level; however, rival firm $j$ responds to the initial input price selected by firm $i$ by adjusting his price-cost margin. In equilibrium, firm $j$ selects the wider price-cost margin of the PTS game when $\Theta < 0$. The wider equilibrium price-cost margin that emerges for the Bertrand player in the (PTS, Bertrand) subgame subsequently increases the volume of sales for the PTS player at the Bertrand equilibrium price-cost margin, making the choice of PTS a facilitating practice. The opposite is true –defecting from Bertrand to the PTO subgame is a facilitating practice– when $\Theta > 0$.

Now consider the outcome for profits under the system of equations (16) and (17). Evaluating the symmetric pay-off matrix in the first stage of the game yields:

**Proposition 4** *Under linear supply and demand conditions:*

**(i)** *(Bertrand,Bertrand) is an equilibrium if and only if* $\Theta = 0$*;*

**(ii)** *(PTS,PTS) is the unique equilibrium when* $\Theta < 0$*; and*

**(iii)** *(PTO,PTO) is the unique equilibrium when* $\Theta > 0$.

The intuition for this result is that the choice of timing allows firms to employ prices in the relatively less rivalrous market to serve as a facilitating practice to soften price competition in the more rivalrous market. When $\Theta < 0$, a firm that chooses to play PTS causes his rival to adjust his price-cost margin upward in equilibrium, because $k^s > 1$. Moreover, as demonstrated by the entries in Table 1, this adjustment occurs irrespective of the timing of price-setting behavior chosen by the rival firm. The larger price-cost margin selected by the rival firm benefits the firm playing PTS, and as a result, both firms select PTS in the first stage of the game. A comparable outcome occurs when $\Theta > 0$, which allows both firms to coordinate on PTO.

Before examining outcomes with inventory-holding behavior, it is worthwhile to consider the case in which input prices are strategically independent in the upstream market, $\epsilon_s = 0$. When $\epsilon_s = 0$, (PTS, PTS) is the unique equilibrium outcome in the linear case and the margin adjustment by each firm is $k^s = 1 + (D_i^j)^2/\Delta$. The unique equilibrium outcome of the game is Cournot. If the firms instead engaged in PTO, $k^o$ reduces in this case to $k^o = 1$, and the PTO equilibrium would produce the Bertrand outcome. As in Kreps and Scheinkman (1983), the firms would wish to defect from selecting output prices and receiving Bertrand profits by first committing to the purchase of stocks ("capacity"), which results in the Cournot outcome.

## 4.2 Inventory-Holding

Now consider a setting in which inventory-holding behavior is possible. For expositional clarity, we limit our attention to market conditions that satisfy $\Theta < 0$, which provides strategic incentives for PTS to emerge as the unique equilibrium outcome in the case where holding inventory is not possible. Analogous conditions apply for PTO in settings where forward contracts do not represent binding commitments to deliver finished goods to consumers in the downstream market.[4]

Suppose stocks procured from the upstream input market in the PTS game can be freely disposed. The possibility of free disposal of inputs relaxes the inventory constraint in equation (3), which becomes $D^i(\mathbf{p}) \leq S^i(\mathbf{w})$ for $i = 1, 2$.

To study the pricing subgame with fixed supplies and potentially slack inventory constraints, we follow Kreps and Scheinkman (1983) and assume that residual demand is efficiently rationed. We then characterize the profitability of defection strategies from the no-inventory equilibrium using the technique of Allison and Lepore (2012). This analysis results in the following:

**Proposition 5** *The inventory constraint in equation (3) is always binding when* [5]

$$P_j^i(\mathbf{y}) S_i^j(\mathbf{w}) \leq 1 + \varepsilon_o^s. \tag{18}$$

The implication of Proposition 5 is that the PTS equilibrium described above under the assumption of no inventory-holding coincides with the equilibrium outcome in a general setting with inventory-holding behavior provided that condition (18) holds. The interpretation of this condition is as follows. After input procurement has taken place, the procurement cost of the firm is sunk, leaving the firm with the potential to sell less than the procured quantity with no additional cost under the assumption of free disposal. Proposition 5 describes market conditions under which a firm selecting quantities in a setting with no production cost would wish to select an output level that is (at least weakly) greater than the output level implied by PTS, which results in a binding inventory constraint.

[4] The possibility of holding negative inventory under PTO implicitly assumes that forward contracts with consumers can be renegotiated. In the event that forward contracts are non-renegotiable, free disposal of forward contracts would not be possible, and an additional deterrent would exist to holding inventory under PTO.

[5] In the linear case, all terms in condition (18) are constant and the inequality can be written, $\gamma c(\beta - \gamma) \leq \Delta(2\beta - \gamma)$, which holds for sufficiently "small" $c, \gamma$.

# 5 Mode of Competition

Characterizing and measuring the mode of competition in the oligopoly model is essential for deriving policy prescriptions in settings with strategic pre-commitment. It is also important for deriving inferences on the type of market conditions that warrant antitrust scrutiny, for instance market features that favor the use of slotting allowances as a practice to soften price competition.[6] Our goal in this section is to develop testable hypotheses on the mode of competition in industrial settings.

To characterize the mode of competition in intermediated oligopoly settings, consider the second partials of profit under PTS. Dropping arguments for notational convenience, the effect of an input price change by firm $j$ on the marginal profit of firm $i$ under PTS is

$$\pi_{ij}^{i,s} \equiv (p^i - w_i)S_{ij}^i + (p_i^i - 1)S_j^i + p_j^i S_i^i + p_{ij}^i S^i, \qquad i = 1, 2. \tag{19}$$

Prices may be strategic complements ($\pi_{ij}^{i,s} > 0$) or strategic substitutes ($\pi_{ij}^{i,s} < 0$) in the sense of Bulow, Geanakoplos, and Klemperer (1985).

To provide an intuitive characterization of these outcomes, consider the first-order effects in expression (19). Characterizing the mode of competition in the linear model is important for deriving testable hypotheses on the mode of competition for empirical work that relies on linear estimation techniques. On substitution of equations (2.2) and (9), the mode of competition can be expressed as

$$\pi_{ij}^{i,s} = \left( \frac{D_j^j S_i^i - D_j^i S_i^j}{\Delta} - 1 \right) S_j^i + \left( \frac{D_j^j S_j^i - D_j^i S_j^j}{\Delta} \right) S_i^i.$$

In the symmetric market equilibrium, this condition becomes

$$\pi_{ij}^{s} \overset{s}{=} \frac{(p-w)}{p} \varepsilon_o^d \epsilon_s \left(1 - \epsilon_d^2\right) + \Theta, \tag{20}$$

where the first term on the right hand side of equation (20) is positive and $\Theta < 0$ is negative in the PTS game by Proposition 1. It can be seen immediately upon inspection of terms in equation (20) that prices are strategic complements when markets

[6] The Federal Trade Commission (FTC), which regulates the grocery industry, refused to provide guidelines for slotting allowances, citing the need for further investigation on the efficiency effects of the practice (FTC 2001).

are equally rivalrous, $\Theta = 0$ (Bertrand), whereas prices are strategic substitutes in the case where prices are strategically independent in the upstream market, $\epsilon_s = 0$ (Cournot). In general, $\Theta < 0$ is a necessary but not a sufficient condition for prices to be strategic substitutes under PTS.

In the PTO game, the effect of an output price change by firm $j$ on the marginal profit of firm $i$ is

$$\pi_{ij}^{i,o} = (p_i - w^i - t_i)D_{ij}^i + (1 - w_i^i)D_j^i - w_j^i D_i^i - w_{ij}^i D^i. \tag{21}$$

As in the PTS game, output prices may be strategic complements ($\pi_{ij}^{i,o} > 0$) or strategic substitutes ($\pi_{ij}^{i,o} < 0$) depending on the magnitude of $\Theta > 0$. Confining attention to first-order effects in expression (21) and making substitutions from (12) and (13) in the symmetric market equilibrium yields

$$\pi_{ij}^{o} \overset{s}{=} \frac{(p-w)}{w}\varepsilon_o^s \epsilon_d \left(1 - \epsilon_s^2\right) - \Theta. \tag{22}$$

Under PTO ($\Theta > 0$), the first term on the right-hand side of the expression is positive and the second term is negative. The relative rivalry of the upstream market influences the mode of competition under PTO in the opposite manner as under PTS: The model reduces to Bertrand when $\Theta = 0$ and to Cournot oligopsony when $\epsilon_d = 0$. A relatively rivalrous upstream market ($\Theta > 0$) is necessary but not sufficient for output prices to be strategic substitutes under PTO.

**Proposition 6** *Under conditions of linear supply and demand, prices are strategic substitutes in the intermediated oligopoly model when:*

**(i)** $\epsilon_s \leq \frac{\epsilon_d}{\left[1+\frac{(p-w)}{p}\varepsilon_o^d\left(1-\epsilon_d^2\right)\right]}$*; or*

**(ii)** $\epsilon_d \leq \frac{\epsilon_s}{\left[1+\frac{(p-w)}{w}\varepsilon_o^s(1-\epsilon_s^2)\right]}$.

Under circumstances in which $\Theta < 0$ ($\epsilon_s < \epsilon_d$), the upstream market is relatively less rivalrous than the downstream market. Firms are able to soften downstream price competition by selecting input prices prior to output prices in the PTS game. Part

($i$) of Proposition **??** is the relevant criteria for the mode of competition, and prices are strategic substitutes when $\epsilon_s$ is sufficiently small relative to $\epsilon_d$. The opposite is true when $\Theta > 0$ ($\epsilon_d < \epsilon_s$). Part ($ii$) of Proposition **??** is the relevant criteria for the mode of competition, and prices are strategic substitutes when $\epsilon_d$ is sufficiently small relative to $\epsilon_s$. In either case, the mode of competition is determined by the relative rivalry of the upstream market.

# 6 Policy Implications

The mode of competition in the oligopoly model has important implications in a number of policy settings. In this section, we highlight policy implications of the model for the case of contract design between a principal (a domestic trade authority, a firm, or a controlling shareholder) and an agent (a domestic firm, a supplier/consumer, or a manager). For expositional clarity, we consider strategic policies by the principal that tax or subsidize input procurement by the agent in the upstream market under conditions of linear supply and demand functions. This allows us to numerically compute the optimal tax level for variation in the relative rivalry of the upstream market by perturbing $c$ and $\gamma$ in the system of equations (16) and (17).

We consider the following three-stage game. In stage 1, the principal of firm $i$ imposes a unit tax $t_i$ on inputs procured in the upstream market by agent $i$. In stage 2, agents take principals' policy decisions parametrically and select the timing of pricing decisions, and in stage 3, agents select prices in the relevant subgame.

Letting $\pi^i(\mathbf{p}, \mathbf{w})$ denote the profit of principal $i$, we denote agent $i$'s profit

$$\Pi^i(\mathbf{p}, \mathbf{w}) = \pi^i(\mathbf{p}, \mathbf{w}) - t_i S^i(\mathbf{w}) + \Omega_i, \qquad i = 1, 2, \tag{23}$$

where $\Omega_i$ is a lump-sum transfer that subsumes fixed costs. Maximizing the agent's profit gives the first-order condition

$$\Pi_i^{i,s} \equiv \left(p^i(\mathbf{w}) - w_i - t_i\right) S_i^i(\mathbf{w}) + (p_i^i(\mathbf{w}) - 1) S^i(\mathbf{w}) = 0, \qquad i = 1, 2, \tag{24}$$

for the PTS game and

$$\Pi_i^{i,o} \equiv (p_i - w^i(\mathbf{p}) - t_i) D_i^i(\mathbf{p}) + (1 - w_i^i(\mathbf{p})) D^i(\mathbf{p}) = 0, \qquad i = 1, 2, \tag{25}$$

for the PTO game.

**Proposition 7** *The choice of PTO or PTS as an equilibrium strategy of agent i is invariant to the principal's choice of policy variable $t_i$. Thus, the market equilibrium is Bertrand if $\Theta = 0$, PTO if $\Theta > 0$, and PTS if $\Theta < 0$.*

The magnitudes of the policy variables, $t_i$, $i = 1, 2$ influence neither the sign nor the value of $\Theta$. An implication of Proposition **??** is that our observations on strategic pre-commitment devices are robust to different policy structures that alter the marginal returns facing oligopoly agents.

Confining attention to first-order effects, the second partial of $\Pi^i$ with respect to $w_i$ and $w_j$ is

$$\Pi^{i,s}_{ij} = p^i_j(\mathbf{w})S^i_i(\mathbf{w}) + (p^i_i(\mathbf{w}) - 1)S^i_j(\mathbf{w}) = \pi^{i,s}_{ij}, \qquad i = 1, 2,$$

for the PTS game and the second partial of $\Pi^i$ with respect to $p_i$ and $p_j$ is

$$\Pi^{i,o}_{ij} = (1 - w^i_i(\mathbf{p}))D^i_j(\mathbf{p}) - w^i_j(\mathbf{p})D^i_i(\mathbf{p}) = \pi^{i,o}_{ij}, \qquad i = 1, 2,$$

In the symmetric market equilibrium, it is straightforward to show that the optimal values of the policy variables satisfy

$$t^*_i \overset{s}{=} \pi^{i,s}_{ij} \overset{s}{=} \pi^{i,o}_{ij}, \qquad i = 1, 2.^7 \tag{26}$$

The optimal value of the policy variable for each agent depends on the mode of competition between oligopoly firms. Taxes ($t^*_i > 0$) are optimal when prices are strategic complements, whereas subsidies ($t^*_i < 0$) are optimal when prices are strategic substitutes.

The intermediated oligopoly model allows the optimal value of the strategic pre-commitment policy to be explicitly computes according to supply and demand conditions facing firms. We illustrate this outcome for the case of linear supply and demand functions by calculating the (unique) Nash equilibrium in policy variables

[7] Full derivation of the optimal policy varibles is provided in the Web Appendix.

for variation in the degree of pricing interdependence in each market, $c$ and $\gamma$. For our numerical analysis, we select parameters $a = b = \beta = 1$ and identify the optimal tax policy in the symmetric pre-commitment equilibrium, $t^* = t_i^* = t_j^*$ for variations in $c \in (0, 1]$ and $\gamma \in (0, 1]$.

Figure 1 depicts the contour lines of the symmetric trade policy equilibrium in the space $(c, \gamma)$. Note that PTS is the unique equilibrium of the game when $\gamma > c$, while PTO is the unique equilibrium of the game when $\gamma < c$. The optimal trade policy is a tax ($t^* > 0$) in the shaded region of the figure and a subsidy ($t^* < 0$) in the non-shaded area of the figure. The contour line where the laissez-faire outcome is optimal ($t^* = 0$) is represented by the contour line that separates these two areas. For a fixed $\gamma$ (respectively $c$) the optimal policy reveals an inverted $u$-shaped pattern as $c$ (respectively $\gamma$) increases from 0 to 1.

# 7 Concluding Remarks

The intermediation model results in testable hypotheses on the mode of competition that can be used to derive policy implications under oligopoly. Our analysis reveals the prevailing mode of competition in a particular industry to be an empirical question that depends on estimable parameters of supply and demand functions in the upstream and downstream markets in which firms interact.

We have demonstrated that oligopoly firms have an incentive to select input prices prior to choosing output prices in the PTS game when the downstream market is relatively rivalrous, but to select output prices prior to choosing input prices in the PTO game when the upstream market is relatively rivalrous. Under reasonably mild conditions, we have shown these outcomes to be robust to the possibility of strategic inventory-holding behavior of firms. For both the PTS and PTO games, prices are strategic complements when the upstream market and the downstream market are relatively equally rivalrous, whereas prices are strategic substitutes under circumstances where the upstream (downstream) market is sufficiently rivalrous relative

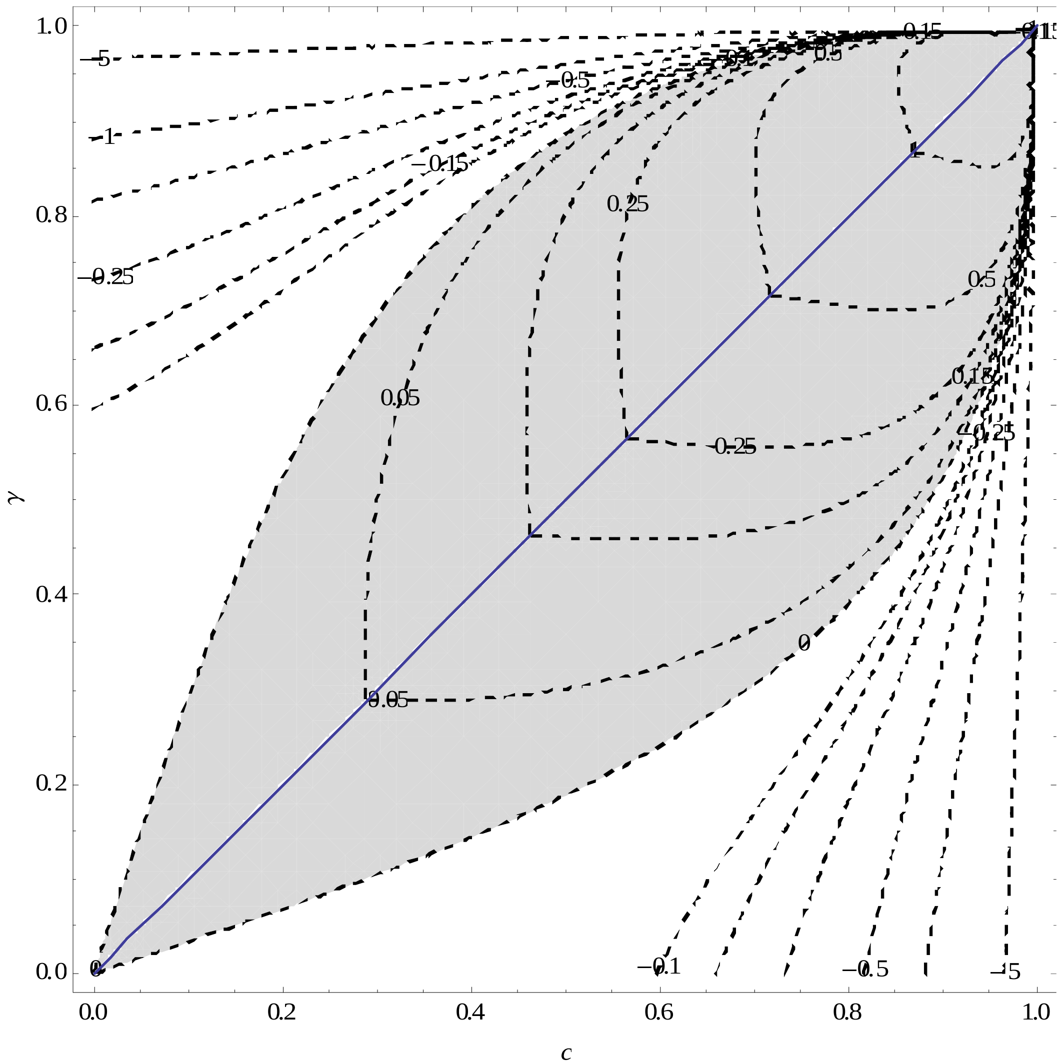


Figure 1:

to the remaining market. In the case where the markets are equally rivalrous, the equilibrium outcome in Bertrand (Nash-in-margins) and in cases where prices are strategically independent in either the upstream market or the downstream market, the equilibrium outcome is Cournot.

Our model provides empirical direction for examining the mode of competition in industrial settings that can be used to tailor pre-commitment policies to the particular conditions facing firms in a given industry. This feature of the model can provide a useful tool for policy formulation as well as for the design of anti-trust regulation.

An interesting avenue for future research is to consider an active role for firms in contributing to market rivalry through product differentiation. The more oligopoly firms engage in specialization, whether by creating differentiating products from common inputs or by discovering new techniques for producing existing products from specialized inputs, the greater their ability to soften price competition with rivals. Yet, it is also clear from our analysis that relaxing market rivalry is not a globally desirable endeavor, and the reason is that it is the relative degree of market rivalry – not the absolute degree– that is the source of strategic advantage. Indeed, a firm that seeks to specialize input requirements would have a greater ability to soften price competition with rivals when downstream products are standardized than when downstream products are highly differentiated. Our model suggests the potential for "strategic standardization" to occur under circumstances in which inputs (outputs) in the upstream (downstream) market are highly specialized.

# References


[1] Bertrand, Joseph. 1883. "Theorie Mathematique de la Richesse Sociale," *Journal des Savants*: 499-508.

[2] Brander, James A. and Barbara J. Spencer. 1985. "Export Subsidies and International Market Share Rivalry," *Journal of International Economics* 18(1-2): 83-100.

[3] Bresnahan, Timothy F. 1981. "Duopoly Models with Consistent Conjectures," *American Economic Review* 71(5): 934-45.

[4] Bulow, Jeremy I., John D. Geanakoplos, and Paul D. Klemperer. 1985. "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy* 93(3): 488-511.

[5] Cournot, Augustine. 1838. *Recherches sur les Principes Mathematiques de la Theorie des Richesses*. Paris.

[6] Eaton, Jonathan and Gene M. Grossman. 1986. "Optimal Trade and Industrial Policy under Oligopoly," *Quarterly Journal of Economics* 101(2): 383-406.

[7] Fershtman, Chaim and Kenneth L. Judd. 1987. "Equilibrium Incentives in Oligopoly," *American Economic Review* 77(5): 927-40.

[8] Fudenberg, Drew, and Jean Tirole. 1984."The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean-and-Hungry Look," *American Economic Review* 74(2): 361-68.

[9] Gal-Or, Esther. 1985. "First Mover and Second Mover Advantages," *International Economic Review* 26(3): 649-53.

[10] Kreps, David M. and Jose A. Scheinkman. 1983. "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics* 14(2): 326-37.

[11] Maggi, Giovanni. 1996. "Strategic Trade Policies with Endogenous Mode of Competition," *American Economic Review* 86(2): 237-58.

[12] Martin, Stephen. 1999. "Kreps And Scheinkman With Product Differentiation: An Expository Note," University of Copenhagen, Centre for Industrial Economics Discussion Paper no. 1999-11.

[13] Shaffer, Greg. 1991. "Slotting Allowances and Resale Price Maintenance: A Comparison of Facilitating Practices," *RAND Journal of Economics* 22(1): 120-36.

[14] Singh, Nirvikar and Xavier Vives. 1984. "Price and Quantity Competition in a Differentiated Oligopoly," *RAND Journal of Economics* 15(4): 546-54.

[15] Sklivas, Steven D. 1987. "The Strategic Choice of Managerial Incentives," *RAND Journal of Economics* 18(3): 452–458.

[16] von Stackelberg, Heinrich. 1934. *Market Structure and Equilibrium (Marktform und Gleichgewicht).* 1st Edition Translation into English, Springer: Bazin, Urch & Hill, 2011.

[17] Stahl, Dale O. 1988. "Bertrand Competition for Inputs and Walrasian Outcomes," *American Economic Review* 78(1): 189-201.

[18] Vickers, John. 1985. "Delegation and the Theory of the Firm," *Economic Journal* 95 (Supplement): 138-47.

[19] Vives, X. 1999. *Oligopoly Pricing: Old Ideas and New Tools.* (Cambridge: MIT Press).

# Appendix

## A Proof of proposition 1

The proof is constructed by comparing outcomes under PTO, PTS, and Bertrand-Nash to the monopoly outcome. Our goal is to show that the monopoly prices $(p^M, w^M)$ satisfy $w^M < w^s \leq w^B$ and $p^M > p^o \geq p^B$. When this condition holds, firm profits are rising for defections from $w^B$ $(p^B)$ that involve $w < w^B$ $(p > p^B)$ under PTS (PTO).

A multi-product monopolist chooses a quantity pair $(y_1, y_2)$ to maximize profits,

$$\pi^M(\mathbf{y}) = \sum_{i \in 1,2} \left[ \left( P^i(\mathbf{y}) - W^i(\mathbf{y}) \right) y_i - F_i \right],$$

where $P^i(\mathbf{y})$ is inverse demand and $W^i(\mathbf{y})$ is inverse supply for product $i$. The first-order necessary condition is

$$\pi_i^M(\mathbf{y}) \equiv P^i - W^i + \left( \frac{\partial P^i}{\partial y_i} - \frac{\partial W^i}{\partial y_i} \right) y_i + \left( \frac{\partial P^j}{\partial y_i} - \frac{\partial W^j}{\partial y_i} \right) y_j = 0, \qquad i = 1, 2, i \neq j \tag{27}$$

Simultaneously solving (27) in the case of a symmetric equilibrium yields the equilibrium quantity, $y^M (= y_1^M = y_2^M)$, which can be used to recover the monopoly prices.

To facilitate the comparison of the monopoly outcome with the Bertrand, PTS, and PTO equilibria, note that $\frac{\partial P^i}{\partial y_i} = \frac{D_j^j}{\Delta} < 0$, $\frac{\partial P^j}{\partial y_i} = \frac{-D_j^i}{\Delta} < 0$, $\frac{\partial W^i}{\partial y_i} = \frac{S_j^j}{\Sigma} > 0$ and $\frac{\partial W^j}{\partial y_i} = \frac{-S_j^i}{\Sigma} > 0$. Incorporating these terms in (27) and evaluating the slope of the monopoly profit function at the Bertrand price level, $(p^B - w^B) = \left( \frac{S^i}{S_i^i} - \frac{D^i}{D_i^i} \right)$, gives

$$\pi_i^M(p^B, w^B) = y \left[ \frac{S_i^j (S_i^i - S_j^i)}{S_i^i \Sigma} + \frac{D_i^j (D_j^i - D_i^i)}{D_i^i \Delta} \right] < 0.$$

Hence the symmetric monopoly output level satisfies $y^M < y^B$. By the concavity of the profit expression, it follows immediately that $w^M < w^B$ and $p^M > p^B$.

It remains to compare the monopoly solution to the equilibrium outcomes under PTS and PTO. Our conjecture is that profitable defections from the Bertrand-Nash equilibrium involve $w < w^B$ and $p > p^B$; hence, firms earn positive rents from playing PTO if $\Theta > 0$ and earn positive rents from playing PTS if $\Theta < 0$. This conjecture holds provided that $w^M < w^s < w^B$ and $p^M > p^o > p^B$. To verify this, consider the slope of monopoly profit at the symmetric PTS equilibrium position, which is

$$\pi_i^M(p^s, w^s) = \frac{y}{S_i^i}\left[1 - \frac{(D_j^j S_i^i - D_j^i S_i^j)}{\Delta}\right] + y\left(\frac{D_j^j}{\Delta} - \frac{S_j^j}{\Sigma} - \frac{D_i^j}{\Delta} + \frac{S_i^j}{\Sigma}\right).$$

Factoring this expression yields

$$\pi_i^M(p^s, w^s) = \frac{y\left(S_i^i - S_j^i\right)}{S_i^i}\left[\frac{S_i^j}{\Sigma} - \frac{D_i^j}{\Delta}\right] < 0.$$

The symmetric monopoly output level satisfies $y^M < y^s$, and it follows by the concavity of the monopoly profit expression that $w^M < w^s$ and $p^M > p^s$.

Proceeding similarly in the case of PTO competition, evaluating the slope of monopoly profit at the symmetric PTO equilibrium position gives

$$\pi_i^M(p^o, w^o) = \frac{y\left(D_i^i - D_j^i\right)}{D_i^i}\left[\frac{S_i^j}{\Sigma} - \frac{D_i^j}{\Delta}\right] < 0.$$

By inspection, $y^M < y^o$, so that $w^M < w^o$ and $p^M > p^o$ in the PTO game. □

# B Proof of Proposition 2

First consider PTS. Note that $\frac{\partial P^i}{\partial y_i} = \frac{D_j^j}{\Delta} < 0$ and $\frac{\partial P^i}{\partial y_i} = \frac{S_j^j}{\Sigma} > 0$. Incorporating these terms in optimality condition (14), and making use of the resulting equation to evaluate the slope of profit expression (8) at the symmetric Cournot equilibrium position $(w^C, p^C)$ gives

$$\pi_i^{i,s}(w^C, p^C) = S^i S_j^i\left(\frac{S_j^i}{\Sigma} - \frac{D_j^i}{\Delta}\right) \leq 0.$$

The term in brackets is negative by our stability conditions. By inspection, the Cournot outcome emerges under PTS ($y^s = y^C$) when $S_j^i = 0$ ($\epsilon_s = 0$).

Proceeding similarly in the case of PTO competition, evaluating the slope of the profit expression (11) at the symmetric Cournot equilibrium position $(w^C, p^C)$ gives

$$\pi_i^{i,o}(w^C, p^C) = D^i D_j^i \left( \frac{S_j^i}{\Sigma} - \frac{D_j^i}{\Delta} \right) \leq 0.$$

By inspection, $y^o = y^C$ when $D_j^i = 0$ ($\epsilon_d = 0$). $\square$

# C Proof of Proposition 3

The proof is constructed by sequentially evaluating the asynchronous pricing outcomes in the stage 2 subgame.

**Lemma 1.** *When firm $i$ plays PTS and firm $j$ plays PTO the equilibrium price-cost margins are $p^i - w_i = \frac{S^i}{S_i^i} k^o - \frac{D^i}{D_i^i}$ and $p_j - w^j = \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j} k^s$, where $k^o = 1 - \frac{S_j^i S_j^j}{\Sigma}\Theta$ and $k^s = 1 + \frac{D_i^j D_j^j}{\Delta}\Theta$.*

**Proof.** Totally differentiating the binding inventory constraints yields

$$\begin{bmatrix} D_i^i & -S_j^i \\ D_i^j & -S_j^j \end{bmatrix} \begin{bmatrix} dp_i \\ dw_j \end{bmatrix} = \begin{bmatrix} S_i^i & -D_j^i \\ S_i^j & -D_j^j \end{bmatrix} \begin{bmatrix} dw_i \\ dp_j \end{bmatrix},$$

and the corresponding responses

$$\frac{\partial p^i}{\partial w_i} = \frac{-S_j^j S_i^i + S_j^i S_i^j}{-S_j^j D_i^i + S_j^i D_i^j} = -\frac{\Sigma}{\Psi} < 0$$

$$\frac{\partial w^j}{\partial w_i} = \frac{S_i^j D_i^i - S_i^i D_i^j}{-S_j^j D_i^i + S_j^i D_i^j} = -\frac{D_j^j S_j^j \Theta}{\Psi} \overset{s}{=} \Theta$$

$$\frac{\partial p^i}{\partial p_j} = \frac{S_j^j D_j^i - S_j^i D_j^j}{-S_j^j D_i^i + S_j^i D_i^j} = \frac{D_i^i S_i^i \Theta}{\Psi} \overset{s}{=} -\Theta$$

$$\frac{\partial w^j}{\partial p_j} = \frac{-D_j^j D_i^i + D_j^i D_i^j}{-S_j^j D_i^i + S_j^i D_i^j} = -\frac{\Delta}{\Psi} < 0$$

where $\Psi = -S_j^j D_i^i + S_j^i D_i^j > 0$, $\Sigma = S_j^j S_i^i - S_j^i S_i^j > 0$, $\Delta = D_j^j D_i^i - D_j^i D_i^j > 0$ and using symmetry, $\Theta = \frac{S_i^i D_i^j - S_i^j D_i^i}{D_j^j S_j^j}$.

Next, consider the problems of firms $i$ and $j$ in stage 1. The profit function for firm $i$ is

$$\max_{w_i} \pi^{i,s} = \left(p^i(w_i, p_j) - w_i\right) S^i(w_i, w^j(w_i, p_j)),$$

with the corresponding first-order necessary condition

$$\left(p^i(w_i, p_j) - w_i\right) \left(S_i^i + S_j^i \frac{\partial w^j}{\partial w_i}\right) + \left(\frac{\partial p^i}{\partial w_i} - 1\right) S^i = 0.$$

Factoring this expression and making use of the inventory constraint gives

$$p^i - w_i = \frac{S^i}{S_i^i} \left(\frac{S_i^i}{\tilde{S}_i^i}\right) - \frac{D^i}{D_i^i} \left(-\frac{D_i^i \Sigma}{\tilde{S}_i^i \Psi}\right)$$

with $\tilde{S}_i^i = S_i^i - S_j^i \frac{D_j^j S_j^j \Theta}{\Psi}$. Expanding terms, we have

$$-\frac{D_i^i \Sigma}{\tilde{S}_i^i \Psi} = \frac{-D_i^i \left(S_j^j S_i^i - S_j^i S_i^j\right)}{S_i^i \left(-S_j^j D_i^i\right) + S_j^i \left(S_i^j D_i^i\right)} = 1$$

and

$$\frac{S_i^i}{\tilde{S}_i^i} = \frac{S_i^i \Psi}{D_i^i \left(S_j^i S_i^j - S_i^i S_j^j\right)} = -\frac{S_i^i \Psi}{D_i^i \Sigma} = 1 - \frac{S_j^i S_j^j}{\Sigma} \Theta.$$

Substitution of terms yields the equilibrium price-cost margin of firm $i$:

$$p^i - w_i = \frac{S^i}{S_i^i} k^o - \frac{D^i}{D_i^i}.$$

The profit function of firm $j$ is

$$\max_{p_j} \pi^{j,o} = \left(p_j - w^j(w_i, p_j)\right) D^j \left(p^i(w_i, p_j), p_j\right)$$

The corresponding first-order necessary condition is

$$\left(p_j - w^j(w_i, p_j)\right) \left(D_j^j + D_i^j \frac{\partial p^i}{\partial p_j}\right) + \left(1 - \frac{\partial w^j}{\partial p_j}\right) D^j = 0,$$

which can be written on substitution of terms as

$$
\begin{aligned}
p_j - w^j &= \frac{-D^j\left(\frac{\Delta}{\Psi}+1\right)}{D_j^j + D_i^j \frac{D_i^i S_i^i \Theta}{\Psi}} \\
&= \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j}\left(-\frac{D_j^j \Psi}{S_i^i \Delta}\right) \\
&= \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j}\left(1+\frac{D_i^j D_j^j \Theta}{\Delta}\right) \\
&= \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j} k^s
\end{aligned}
$$

■

**Lemma 2.** *When firm $i$ plays Bertrand and firm $j$ plays PTS the equilibrium price-cost margins are $p_i - w_i = \frac{S^i}{S_i^i} - \frac{D^i}{D_i^i} k^s$ and $p^j - w_j = \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j}$, where $k^s = 1 + \frac{D_i^j D_j^j}{\Delta}\Theta$.*

**Proof.** Totally differentiating the inventory constraint of firm $j$

$$D_i^j dp_i + D_j^j dp_j = S_i^j dw_i + S_j^j dw_j$$

Bertand timing by firm $i$ yields the following price responses

$$
\begin{aligned}
\frac{\partial p^j}{\partial p_i} &= -\frac{D_i^j}{D_j^j} < 0 \\
\frac{\partial p^j}{\partial w_i} &= \frac{S_i^j}{D_j^j} > 0 \\
\frac{\partial p^j}{\partial w_j} &= \frac{S_j^j}{D_j^j} < 0.
\end{aligned}
$$

Next, consider the problems of firms $i$ and $j$ in stage 1. Firm $i$ chooses $p_i$ and $w_i$ to

$$
\begin{gathered}
\max_{p_i, w_i} p_i D^i(p_i, p^j(p_i, w_i, w_j)) - w_i S^i(w_i, w_j) \\
\text{s.t. } S^i(w_i, w_j) = D^i(p_i, p^j(p_i, w_i, w_j))
\end{gathered}
$$

Letting $\lambda_i$ denote the multiplier of firm $i$, we have the following first-order necessary conditions

$$\begin{aligned} D^i + (p_i - \lambda_i)\left(D_i^i + D_j^i \frac{\partial p^j}{\partial p_i}\right) &= 0 \\ (p_i - \lambda_i)\left(D_j^i \frac{\partial p^j}{\partial w_i}\right) - (w_i - \lambda_i) S_i^i - S^i &= 0 \end{aligned}$$

Using these equations to eliminate $\lambda_i$, we get

$$p_i - w_i = \frac{S^i}{S_i^i} - \frac{D^i}{\hat{D}_i^i} \frac{\hat{S}_i^i}{S_i^i}$$

where

$$\begin{aligned} \hat{S}_i^i &= S_i^i - S_i^j \frac{D_j^i}{D_j^j} = -\frac{\Psi}{D_j^j} < S_i^i \\ \hat{D}_i^i &= D_i^i - D_j^i \frac{D_i^j}{D_j^j} = \frac{\Delta}{D_j^j} > D_i^i \end{aligned}$$

Substitution of terms yields

$$p_i - w_i = \frac{S^i}{S_i^i} - \frac{D^i}{D_i^i}\left(-\frac{\Psi D_i^i}{\Delta S_i^i}\right) = \frac{S^i}{S_i^i} - \frac{D^i}{D_i^i} k^s$$

Next, consider the stage 1 profit function of firm $j$. Firm $j$ chooses $w_j$ to

$$\max_{w_j} (p^j(p_i, w_i, w_j) - w_j) S^j(w_i, w_j),$$

The associated first-order necessary condition for a maximum is

$$(p^j - w_j) S_j^j + \left(\frac{\partial p^j}{\partial w_j} - 1\right) S^j = 0$$

Substiting the stage 2 price response into this condition yields

$$p^j - w_j = \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j}.$$

■

**Lemma 3.** *When firm i plays Bertrand and firm j plays PTO the equilibrium price-cost margins are* $p_i - w_i = \frac{S^i}{S_i^i} k^o - \frac{D^i}{D_i^i}$ *and* $p^j - w_j = \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j}$*, where* $k^o = 1 - \frac{S_j^i S_j^j}{\Sigma} \Theta$.

**Proof.** Totally differentiating the inventory constraint for firm $j$, Bertand timing by firm $i$ yields the following price responses

$$\begin{aligned}\frac{\partial w^j}{\partial p_i} &= \frac{D_i^j}{S_j^j} < 0\\ \frac{\partial w^j}{\partial p_j} &= \frac{D_j^j}{S_j^j} < 0\\ \frac{\partial w^j}{\partial w_i} &= \frac{-S_i^j}{S_j^j} > 0.\end{aligned}$$

Next, consider the problems of firms $i$ and $j$ in stage 1. Firm $i$ chooses $p_i$ and $w_i$ to

$$\begin{gathered}\max_{p_i,w_i} p_i D^i(p_i,p_j) - w_i S^i(w_i, w^j(p_i,p_j,w_i))\\ \text{s.t. } S^i(w_i, w^j(p_i,p_j,w_i)) = D^i(p_i,p_j)\end{gathered}$$

Letting $\lambda_i$ denote the multiplier of firm $i$, we have the following first-order necessary conditions

$$\begin{aligned}D^i + (p_i - \lambda_i)D_i^i - (w_i - \lambda_i)S_j^i\left(\frac{\partial w^j}{\partial p_i}\right) &= 0\\ -S^i - (w_i - \lambda_i)\left(S_i^i + S_j^i\frac{\partial w^j}{\partial w_i}\right) &= 0\end{aligned}$$

Solving these equations simultaneously yields

$$p_i - w_i = \frac{S^i}{S_i^i}\left(-\frac{\Psi S_i^i}{\Sigma D_i^i}\right) - \frac{D^i}{D_i^i} = \frac{S^i}{S_i^i}k^o - \frac{D^i}{D_i^i}$$

Next, consider the stage 1 profit function of firm $j$. Firm $j$ chooses $p_j$ to

$$\max_{p_j}(p_j - w^j(p_i,p_j,w_i))D^j(p_i,p_j),$$

The associated first-order necessary condition for a maximum is

$$(p_j - w^j)D_j^j + \left(1 - \frac{\partial w^j}{\partial p_j}\right)D^j = 0$$

Substiting the stage 2 price response into this condition yields

$$p_j - w^j = \frac{S^j}{S_j^j} - \frac{D^j}{D_j^j}.$$

■

# D Proof of Proposition 4

For the generic case, the equilibrium profit level of firm $i$ is described by four equations

$$\begin{aligned} \frac{p_i - w_i}{S^i} &= \Gamma_i, \qquad i = 1, 2. \\ D^i &= S^i, \qquad i = 1, 2. \end{aligned}$$

The equilibrium profit level of firm $i$ satisfies $\pi^i = (p_i - w_i)S^i = \Gamma_i (S^i)^2$, where

$$\Gamma_i \in \left\{ \Gamma^o = \frac{k^o}{S_i^i} - \frac{1}{D_i^i}, \Gamma^b = \frac{1}{S_i^i} - \frac{1}{D_i^i}, \Gamma^s = \frac{1}{S_i^i} - \frac{k^s}{D_i^i} \right\}.$$

For linear supply and demand, these terms all reduce to constants. Specifically,

$$\Gamma_i \in \left\{ \Gamma^o = \frac{k^o}{\beta} + \frac{1}{b}, \Gamma^b = \frac{1}{\beta} + \frac{1}{b}, \Gamma^s = \frac{1}{\beta} + \frac{k^s}{b} \right\},$$

where $k^o = \frac{\beta(\beta b - \gamma c)}{b(\beta^2 - \gamma^2)}$ and $k^s = \frac{b(\beta b - \gamma c)}{\beta(b^2 - c^2)}$.

The (symmetric) pay-off matrix is

| | Bertrand | PTS | PTO |
|---|---|---|---|
| Bertrand | $\Gamma^b(S^{bb})^2, \Gamma^b(S^{bb})^2$ | $\Gamma^s(S_1^{bs})^2, \Gamma^b(S_2^{bs})^2$ | $\Gamma^o(S_1^{bo})^2, \Gamma^b(S_2^{bo})^2$ |
| PTS | $\Gamma^b(S_1^{sb})^2, \Gamma^s(S_2^{sb})^2$ | $\Gamma^s(S^{ss})^2, \Gamma^s(S^{ss})^2$ | $\Gamma^o(S_1^{so})^2, \Gamma^s(S_2^{so})^2$ |
| PTO | $\Gamma^b(S_1^{ob})^2, \Gamma^o(S_2^{ob})^2$ | $\Gamma^s(S_1^{os})^2, \Gamma^o(S_2^{os})^2$ | $\Gamma^o(S^{oo})^2, \Gamma^o(S^{oo})^2$ |

where we denote $S_i^{hl}$ the equilibrium output of firm $i$ when firm $i$ plays strategy $h \in \{b, o, s\}$ and firm $j$ plays strategy $l \in \{b, o, s\}$, where $b$ stands for Bertrand, $o$ for PTO and $s$ for PTS. By symmetry, we denote also $S_i^{hh} = S_j^{hh} = S^{hh}$ for any $h$.

Note that $\Theta < 0 \Leftrightarrow \Gamma^s > \Gamma^b > \Gamma^o$ and $\Theta > 0 \Leftrightarrow \Gamma^s < \Gamma^b < \Gamma^o$. The system of equilibrium conditions can be rewritten as

$$\begin{aligned} p_i &= (1 + \beta\Gamma_i)w_i - \gamma\Gamma_i w_j, \qquad i = 1, 2, \\ D^i &= S^i, \qquad i = 1, 2. \end{aligned}$$

Replacing $p_i$ by its value in the last two equations, then solving in $w_i$ and replacing in $S^i$, we get

$$S^i = \frac{A + B\Gamma_j}{C + D\Gamma_i\Gamma_j + E(\Gamma_i + \Gamma_j)}$$

with $A = a(\beta-\gamma)(b+c+\beta+\gamma) > 0$, $B = a(b+c)\Sigma > 0$, $C = \Sigma + \Delta + 2\Psi > 0$, and $D = \Delta\Sigma > 0$, $E = \beta\Delta + b\Sigma > 0$. It follows that

$$\Gamma_j \geq \Gamma_i \Leftrightarrow S^i \geq S^j.$$

We get

$$\begin{aligned}
S^{bb} &= \frac{A + B\Gamma^b}{C + D(\Gamma^b)^2 + 2E\Gamma^b} \\
S^{ss} &= \frac{A + B\Gamma^s}{C + D(\Gamma^s)^2 + 2E\Gamma^s} \\
S^{oo} &= \frac{A + B\Gamma^o}{C + D(\Gamma^o)^2 + 2E\Gamma^o} \\
S_1^{bs} &= S_2^{sb} = \frac{A + B\Gamma^b}{C + D\Gamma^s\Gamma^b + E(\Gamma^s + \Gamma^b)} \\
S_2^{bs} &= S_1^{sb} = \frac{A + B\Gamma^s}{C + D\Gamma^s\Gamma^b + E(\Gamma^s + \Gamma^b)} \\
S_1^{ob} &= S_2^{bo} = \frac{A + B\Gamma^o}{C + D\Gamma^o\Gamma^b + E(\Gamma^o + \Gamma^b)} \\
S_2^{ob} &= S_1^{bo} = \frac{A + B\Gamma^b}{C + D\Gamma^o\Gamma^b + E(\Gamma^o + \Gamma^b)} \\
S_1^{os} &= S_2^{so} = \frac{A + B\Gamma^o}{C + D\Gamma^s\Gamma^o + E(\Gamma^s + \Gamma^o)} \\
S_2^{os} &= S_1^{so} = \frac{A + B\Gamma^s}{C + D\Gamma^s\Gamma^o + E(\Gamma^s + \Gamma^o)}
\end{aligned}$$

For part $(i)$, to verify that (Bertrand,Bertrand) is an equilibrium iff $\Theta = 0$ notice that it must be true in the symmetric case that

$$\Gamma^b(S^{bb})^2 \geq \max(\Gamma^b(S_1^{sb})^2, \Gamma^b(S_1^{ob})^2)$$

or equivalently

$$S^{bb} \geq \max(S_1^{sb}, S_1^{ob})$$

We have

$$\begin{aligned}
S^{bb} - S_1^{sb} &= \frac{A + B\Gamma^b}{C + D(\Gamma^b)^2 + 2E\Gamma^b} - \frac{A + B\Gamma^s}{C + D\Gamma^s\Gamma^b + E(\Gamma^s + \Gamma^b)} \\
&\stackrel{s}{=} \left(A + B\Gamma^b\right)\left(C + D\Gamma^s\Gamma^b + E(\Gamma^s + \Gamma^b)\right) - \left(A + B\Gamma^s\right)\left(C + D(\Gamma^b)^2 + 2E\Gamma^b\right) \\
&\stackrel{s}{=} AC + AD\Gamma^s\Gamma^b + AE(\Gamma^s + \Gamma^b) + BC\Gamma^b + BD(\Gamma^b)^2\Gamma^s + BE(\Gamma^s + \Gamma^b)\Gamma^b \\
&\quad -AC - AD(\Gamma^b)^2 - 2AE\Gamma^b - BC\Gamma^s - BD\Gamma^s(\Gamma^b)^2 - 2BE\Gamma^b\Gamma^s \\
&\stackrel{s}{=} \left(\Gamma^s - \Gamma^b\right)\left(\left(AD - BE\right)\Gamma^b + AE - BC\right) \\
&\stackrel{s}{=} \left(\Gamma^b - \Gamma^s\right)
\end{aligned}$$

because

$$\begin{aligned}
AD - BE &= -a(b + c)\Sigma(c\Sigma + \gamma\Delta) < 0, \\
AE - BC &= -A(c\Sigma + \gamma\Delta) < 0.
\end{aligned}$$

Accordingly,

$$S^{bb} - S_1^{ob} \stackrel{s}{=} \left(\Gamma^b - \Gamma^o\right)$$

Hence, when $\Theta < 0$, we have $S^{bb} < S_1^{sb}$ and when $\Theta > 0$, we have $S^{bb} < S_1^{ob}$. It follows that (Bertrand, Bertrand) is an equilibrium only when $\Theta = 0$, which completes the proof of part $(i)$.

Next consider part $(ii)$. For (PTS,PTS) to be an equilibrium of the game with symmetric firms, it must be true that

$$\Gamma^s(S^{ss})^2 \geq \max(\Gamma^s(S_1^{bs})^2, \Gamma^s(S_1^{os})^2)$$

or equivalently

$$S^{ss} \geq \max(S_1^{bs}, S_1^{os})$$

Following the approach detailed above in part $(i)$,

$$\begin{aligned}
S^{ss} - S_1^{bs} &= \frac{A + B\Gamma^s}{C + D(\Gamma^s)^2 + 2E\Gamma^s} - \frac{A + B\Gamma^b}{C + D\Gamma^s\Gamma^b + E(\Gamma^s + \Gamma^b)} \\
&\stackrel{s}{=} \left(\Gamma^s - \Gamma^b\right)
\end{aligned}$$

and

$$\begin{aligned} S^{ss} - S_1^{os} &= \frac{A + B\Gamma^s}{C + D(\Gamma^s)^2 + 2E\Gamma^s} - \frac{A + B\Gamma^o}{C + D\Gamma^s\Gamma^o + E(\Gamma^s + \Gamma^o)} \\ &\stackrel{s}{=} (\Gamma^s - \Gamma^o) \end{aligned}$$

and the result follows that (PTS,PTS) is the equilibrium of the game when $\Theta < 0$.

The proof of part $(iii)$ follows similar logic as in the (PTS, PTS) case. □

# E Proof of Proposition 5

Consider the PTS game that allows for inventory holding. We begin by describing the equilibrium of the output pricing game when supply is fixed by wage choice in the initial stage of the game. Denote these levels by $x_i = S^i(\mathbf{w})$. Notice that the second stage analysis with fixed supplies $x_i$ is exactly the same as the Bertrand-Edgeworth pricing game with fixed capacities.

Before turning to the Bertrand-Edgeworth pricing game, we need to define two other games that will be useful for our analysis: the zero-cost Bertrand game and the zero-cost Cournot game.

In the zero-cost Bertrand game, each firm $i$ has the profit function

$$D^i(\mathbf{p})p_i = \max\{a - bp_i + cp_j, 0\}p_i,$$

The best response function of firm $i$ is

$$R^i(p_j) = \max\left\{\frac{a + cp_j}{2b}, 0\right\}.$$

In the symmetric market equilibrium, this game has a unique, symmetric Nash equilibrium in pure strategies. We label the symmetric equilibrium price

$$p^b = \frac{a}{2b - c}.$$

In the zero-cost Cournot game, each firm $i$ has the profit function

$$P^i(\mathbf{x})x_i = \max\left\{\frac{1}{b^2 - c^2}\left(a(b + c) - bx_i - cx_j\right), 0\right\}x_i$$

where $\mathbf{x} = (x_1, x_2)$ are the firms' quantities. Each firm has a best response function to the other firm's quantity given by

$$r^i(x_j) = \max\left\{\frac{1}{2b}(a(b+c) - cx_j), 0\right\}.$$

In the symmetric market equilibrium, the unique Nash equilibrium in this game is symmetric in pure strategies, with each firm's quantity given by

$$q^c = \frac{a(b+c)}{2b+c}.$$

The symmetric, Cournot equilibrium price is

$$p^c = \frac{ab}{b(2b-c) - c^2} > p^b$$

### E.1 Bertrand-Edgeworth Pricing Game

Given that residual demand is efficiently rationed, the residual demand of firm $i$ is

$$d^i(p_i, x_j) = \max\left\{\frac{1}{b}\left((b+c)\left(a - (b-c)p_i\right) - cx_j\right), 0\right\}$$

The price that maximizes the residual profit $p_i d^i(p_i, x_j)$ is

$$p^R(x_j) = \frac{1}{2(b-c)}\left(a - \frac{c}{b+c}x_j\right),$$

and the maximum residual profit is

$$\pi^R(x_j) = \frac{b+c}{4b(b-c)}\left(a - \frac{c}{b+c}x_j\right)^2.$$

Now we turn to characterizing the equilibrium of the pricing game with fixed capacities.

#### E.1.1 Cournot Pricing

Without loss of generality we assume that $x_1 \geq x_2$. We separate the analysis of the pricing subgame into two regions of capacities. The first is such that $x_1 \leq r(x_2)$. Note that $x_1 \geq x_2$ and $x_1 \leq r(x_2)$ imply that $x_2 \leq r(x_1)$. In this region of capacities, the pricing subgame has a unique pure strategy Nash equilibrium with market clearing prices.

**Lemma 4.** The unique Nash equilibrium of the of the pricing subgame is $p_1^* = P^1(\mathbf{x})$ and $p_2^* = P^2(\mathbf{x})$ if $x_1 \leq r(x_2)$.

**Proof.** **Step 1.** $p^* = \big(P^1(\mathbf{x}), P^2(\mathbf{x})\big)$ **is a Nash equilibrium**

Take $x_1 \leq r(x_2)$ and $p_j = P^j(\mathbf{x})$. If $p_i \leq P^i(\mathbf{x})$, then the profit of firm $i$ is $p_i x_i$. Clearly, the profit of firm $i$ is maximized at $p_i = P^i(\mathbf{x})$ if we constrain $p_i \leq P^i(\mathbf{x})$. If $p_i \geq P^i(\mathbf{x})$, then the profit of firm $i$ is $p_i \min\{d^i(p_i, x_j), x_i\}$. By the definition of efficient rationing $\max_{p_i} \big\{p_i \min\{d^i(p_i, x_j), x_i\}\big\} = P^i(\mathbf{x})x_i$. This must be true since $x_i \leq r(x_j)$, which implies that $p^R(x_j) = P^i(r(x_j), x_j) \leq P^i(\mathbf{x})$. Thus, the maximum price must be at the boundary where $d^i(p_i, x_j) = x_i$, which is the prices $P^i(\mathbf{x})$. Consequently, if $p_j^* = P^j(\mathbf{x})$, then it is a best response for firm $i$ to pick $p_i^* = P^i(\mathbf{x})$.

**Step 2. Uniqueness of pure strategy equilibrium**

Suppose to the contrary that there is another equilibrium $\mathbf{p}^o$. We will show a contradiction.

*Case i.* There is a firm $i$ such that $D^i(\mathbf{p}^o) > x_i$. Firm $i$ gains profit by picking $p_i$ greater such that $D^i(p_i, p_j^o) \geq x_i$.

*Case ii.* There is an $i$ such that $D^i(\mathbf{p}^o) < x_i$ for at least one firm $i \in \{1, 2\}$.

Suppose that the other firm $j$ is also pricing such that $D^j(\mathbf{p}^o) < x_j$. If this is true the profits of each firm are the same as in the zero cost Bertrand game at $\mathbf{p}^o$. Since there is no zero cost Bertrand equilibrium price such that $D^j(\mathbf{p}^b) < x_j$ for both firms $j \in \{1, 2\}$ when $x_i \leq r(x_j)$, we know that $\beta(p_j^o) < p_j^o$ for $j \in \{1, 2\}$. Thus, either $\beta(p_j^o)$ or $P^j(\mathbf{x})$ leads to higher profit than $p_i^o$.

Suppose that the other firm $j$ is pricing such that $D^j(\mathbf{p}^o) = x_j$. Then firm $i$ is operating on the residual demand, which is uniquely maximized at $P^i(\mathbf{x})$.

*Case iii.* $D^i(\mathbf{p}^o) = x_i$ for both firms $i$. Then at least one firm $i$ must be playing $p_i^o < P^i(\mathbf{x})$. The other firm $j$'s best response is to maximize the residual demand by which is uniquely done by picking $P^j(\mathbf{x})$. So $p_j^o = P^j(\mathbf{x})$. Consequently, firm $i$'s unique best response to $P^i(\mathbf{x})$ is to maximize the residual profit with $P^i(\mathbf{x})$.

**Step 3. Uniqueness of equilibrium**

The argument is based on showing that if $x_1 \leq r(x_2)$, then the game has *strategic complementaries* and then appealing to Theorem 12 in Milgrom and Shannon (1994). Adapting Milgrom and Shannon definition of a game with *strategic complementarities* to our notation, we have: for every player $i$

1. Player $i$'s strategy space is a compact lattice;

2. Player $i$'s payoff function is upper semi-continuous in $p_i$ for $p_j$ fixed, and continuous in $p_j$ for fixed $p_i$;

3. Player $i$'s payoff function is quasisupermodular in $p_i$ and satisfies the single crossing property in $(p_i; p_j)$.

The first two requirements are satisfied for the pricing game, since the interval $[0, P] \subset \mathbb{R}$ is a compact lattice and the profits are continuous functions in both arguments. We are left to show that property three is satisfied.

The connection between monotonic nondecreasing best responses and the third requirement for a game with strategic complementarities is based on the following special case of Theorem 4 in Milgrom and Shannon (1994) modified to our notation.

[Milgrom and Shannon] Let $\pi : [0, P] \times [0, P] \mapsto \mathbb{R}$. Then $\phi(\rho)$ is monotonic nondecreasing in $\rho$ if and only if $\pi$ is quasisupermodular in $p$ and satisfies the single crossing property in $(p; \rho)$.

Now we appeal to a special case of Milgrom and Shannon (1994), Theorem 12 modified to our notation:

[Milgrom and Shannon] In a game with strategic complementarities and a unique pure strategy Nash equilibrium, the Nash equilibrium strategies are the unique serial undominated strategies.

We will show that each firm's best response is a nondecreasing function on $[0, P]$ and then uniqueness follows from Fact 2. To do this, we show that the firms' best response correspondences are contained within monotonic nondecreasing correspondence that have the unique pure strategy equilibrium $\big(P^1(\mathbf{x}), P^2(\mathbf{x})\big)$. Denote by $\boldsymbol{\beta}^{\mathbb{C}}(\mathbf{p})$ the best response correspondence for the pricing subgame.

Define another correspondence

$$\widetilde{\beta}_j^{\mathbb{C}}(p) = \begin{cases} [\beta(p), p^R(x_j)] & p < p^R(x_j) \\ [p^R(x_j), \beta(p)] & p \geq p^R(x_j) \end{cases} .$$

$\widetilde{\beta}_i^{\mathbb{C}}$ is a monotonic nondecreasing correspondence on $[0, P]$ and is such that $\beta_i^{\mathbb{C}}(p) \in \widetilde{\beta}_i^{\mathbb{C}}(p)$ for all $p \in [0, P]$ for either firm $i$. Further, the unique pure strategy equilibrium of the game with best responses $\widetilde{\boldsymbol{\beta}}^{\mathbb{C}}$ is $\big(P^1(\mathbf{x}), P^2(\mathbf{x})\big)$. ■

### E.1.2 Mixed Pricing

In this section, we only prove what is necessary for the proof of our main proposition about the PTS game with inventory-holding. We only need to consider $x_2 \leq q^c$ and without loss of generality $x_1 \geq x_2$. We will use the following nomenclature throughout the proofs of this subsection: As denoted previously, the maximal *residual profit* is $\pi^R(x_j)$, we will refer to this as simply the residual profit. We denote the *front-side profit* of firm $i$ as the largest profit such that the firm $j$ is best off taking the residual demand.

**Lemma 5.** If $x_1 > r(x_2)$, then $\pi_1^* \geq \pi^R(x_2)$

**Proof.** By picking $p_1 = p_1^R(x_2)$ can always get at least the residual demand $d^1(p_1, x_2)$ and hence at least the profit $\pi^R(x_2)$. ■

**Lemma 6.** Firm 1's front-side profit is less than its residual profit.

**Proof.** Take $x_1 \geq x_2$ and $r(x_2) = x_1$ (note that it must be that $x_2 \leq q^c$). This is the Cournot pricing region characterized in Lemma E.1.1. In the Cournot pricing region the front-side profit equals the residual profit, which equals the Cournot profit. Now we will show an increase to $x_1'$ such that $x_1' > r(x_2)$ decreases firm 1's front-side profit while not affecting firm its residual profit. First notice that the residual profit $\pi^R(x_2)$ is not affected by increasing $x_1$. Now we show the front-side profit is decreasing in $x_1$. The front-side profit for firm 1 is bounded above by $\rho_1^f x_1$ where $\rho_1^f = \max\{0, p_1^f\}$ and $D^2(\widetilde{p}_2, p_1^f) = x_2$, where $\widetilde{p}_2 = \pi^R(x_1)/x_2$. We do not need to consider the case that $p_1^f \leq 0$, since in that case the front-side profit is zero, which is clearly less than a positive residual profit. Let us show that $p_1^f x_1$ is decreasing in $x_1$. Using our demand specification and solving for $p_1^f$ we have

$$p_1^f = -\frac{(a - x_2)}{c} + \frac{b}{c}\widetilde{p}_2.$$

We plug in with the functional forms and differentiate the front-side profit of firm 1 with regards to $x_1$ to find

$$-\frac{(a - x_2)}{c} + \frac{b}{cx_2}\pi^R(x_1) + \frac{b}{cx_2}\frac{\partial \pi^R(x_1)}{\partial x_1}x_1 < 0 \tag{28}$$

The first term is negative, since $x_2 \leq q^c < a$. Thus, if we can show that the sum of the second two terms is negative than the whole expression must be negative as well. Plugging in with the functional forms of $\pi^R(x_1)$ and $\partial \pi^R(x_1)/\partial x_1$, (28) reduces to the inequality

$$\begin{aligned}
\frac{b}{cx_2}\left(\frac{b+c}{4b(b-c)}\left(a - \frac{c}{b+c}x_1\right)^2\right) &< \frac{b}{cx_2}\left(\frac{b+c}{2b(b-c)}\left(a - \frac{c}{b+c}x_1\right)\frac{c}{b+c}x_1\right) \\
a - \frac{c}{b+c}x_1 &< \frac{2c}{b+c}x_1 \\
\frac{a(b+c)}{3c} &< x_1.
\end{aligned}$$

The final inequality must be true because to be in $x_1 < r(x_2)$, $x_1 > q^c$ and $q^c > a(b+c)/3c$. ■

**Lemma 7.** For $x_1 > r(x_2)$, $\pi_2^* > \pi^R(x_1)$

**Proof.** Suppose to the contrary that firm 2 does not have an equilibrium profit higher than $\pi^R(x_1)$. Suppose that firm 2 plays $\widetilde{p}_2$, then only if firm 1 plays $p_1 \leq p_1^f$ it gets the front-side profit. We know that $p_1^f x_1 < \pi^R(x_2)$ based on Lemma E.1.2. Take $\epsilon > 0$ and suppose firm 2 prices at $\widetilde{p}_2 + \epsilon$, then firm 1 can price at most $p_1^f + b\epsilon/c$ and get the front-side profit. For small enough $\epsilon > 0$, $p_1^f x_1 + b\epsilon x_1/c < \pi^R(x_2)$. This implies that there exists $p_2' > \widetilde{p}_2$ such that firm 1 will always prefer to maximize the residual demand over undercutting and getting the front-side profit. Consequently, firm 2 can guarantee itself a profit greater than maximizing the residual by picking such a $p_2'$, since $p_2' x_2 > \widetilde{p}_2 x_2 = \pi^R(x_1)$. ■

**Lemma 8.** For $x_1 > r(x_2)$, $\pi_1^* \leq \pi^R(x_2)$.

**Proof.** Define $\overline{p}_i$ as the least upper bound of the equilibrium best responses of any firm $i$. The key aspect of this price $\overline{p}_i$ is: Does firm $i$ get residual for sure at $\overline{p}_i$? Note that at least one firm $i$ must get residual for sure at their $\overline{p}_i$. Denote by $\Pr_i(p)$ the equilibrium probability that firm $i$ does not get the residual demand at the price $p$.

The following cases can overlap.

*Case i.* At $\overline{p}_1$ firm 1 gets residual for sure. Consider the case that firm 1 has an atom at $\overline{p}_1$ or firm 1 has best responses approaching $\overline{p}_1$. If firm 1 has an atom at $\overline{p}_1$, then $\pi_1^* = \pi^R(x_2) \geq \overline{p}_1 d^1(\overline{p}_1, x_2)$. If firm 1 has best responses approaching $\overline{p}_1$, then and take any sequence of best responses $p_1^k \uparrow \overline{p_1}$ and $\lim_{k\to\infty}\{\Pr_1(p_1^k)p_1^k x_1 + (1 - \Pr_1(p_1^k))p_1^k d^1(p_1^k, x_2)\} = \overline{p}_1 d^1(\overline{p}_1, x_2) \leq \pi^R(x_2)$.

*Case ii.* At $\overline{p}_2$ firm 2 gets residual for sure. Consider the case that firm 2 has an atom at $\overline{p}_1$ or firm 2 has best responses approaching $\overline{p}_2$. We show that this case cannot be. If firm 2 has an atom at $\overline{p}_2$, then $\pi_2^* = \pi^R(x_2) \geq \overline{p}_2 d^2(\overline{p}_2, x_2)$, which contradicts Lemma E.1.2. Take any sequence of best responses $p_2^k \uparrow \overline{p}_2$, then $\lim_{k\to\infty}\{\Pr_2(p_2^k)p_2^k x_2 + (1 - \Pr_2(p_2^k))p_2^k d^2(p_2^k, x_1)\} = \overline{p}_2 d^2(\overline{p}_2, x_1) \leq \pi^c(x_1)$. But, again, based on Lemma E.1.2 we know that $\pi_2^* > \pi^R(x_1)$, a contradiction. ■

We now turn to our primary results on inventory-holding behavior in the PTS game.

## E.2 PTS equilibrium with inventory holding

The PTS equilibrium wage $\widehat{w}$ is such that

$$S^i(\widehat{\mathbf{w}}) \leq q^c \tag{29}$$

Alternatively, we can define $w^c$ such that $S^i(\mathbf{w}^c) = q^c$ and rewrite Condition 29 as

$$P_j^i(\mathbf{q}^c)S_j^j(\mathbf{w}^c) \leq 1 + \varepsilon_o^S(\mathbf{w}^c) \tag{30}$$

We wish to show that if Condition E.2 holds, then the Nash equilibrium outcome in the PTS game with a binding inventory constraint is a Nash equilibrium outcome of the PTS game with inventory holding.

**Proof.** If $S^i(\widehat{\mathbf{w}}) \leq q^c$, then only defections such that $S^i(w_i', \widehat{w}_j) > r(S^j(\widehat{w}_j, w_i'))$ are potentially more profitable in the PTS game with inventory holding than the PTS game. Any other defection results in exactly the same profit as the PTS game. We first show that the profit of firm $i$ is strictly concave over all $w_i' \geq \widehat{w}_i$ such that $S^i(w_i', \widehat{w}_j) > r(S^j(\widehat{w}_j, w_i'))$ by showing using the fact that the profit is twice continuously differentiable.

$$\frac{\partial^2 \left(\pi^R(S^2) - w_1 S^1\right)}{\partial w_1^2} = \frac{-\gamma c^2}{2b(b^2 - c^2)} - \beta < 0. \tag{31}$$

Thus, the profit of firm 1 for all $S^i(w_i', \widehat{w}_j) > r(S^j(\widehat{w}_j, w_i'))$ is concave.

At the boundary of $S^i(w_i', \widehat{w}_j) = r(S^j(\widehat{w}_j, w_i')$ is the only point where the profit of firm $i$ is not differentiable. Consequently, for such a defection to be profitable the one at the boundary must be marginally better.

We know at $w_i' \geq \widehat{w}_i$ such that $S^i(w_i', \widehat{w}_j) = r(S^j(\widehat{w}_j, w_i')$ that (based on $\widehat{\boldsymbol{w}}$ as an equilibrium of the PTS game)

$$P_j^i(S^i, S^j) S_i^j S^i - S^i - w_i' S_i^i < 0 \tag{32}$$

at such a $w_i'$, it must be that $S^i = r(S^j)$. Thus, the inequality 32 implies that

$$P_j^i(r(S^j), S^j) r(S^j) S_i^j - S^i - w_i S_i^i < 0.$$

Based the strict concavity of the profit for all $w_i' \geq \widehat{w}_i$ such that $S^i(w_i', \widehat{w}_j) > r(S^j(\widehat{w}_j, w_i'))$

$$P_j^i(r(S^j), S^j) r(S^j) S_i^j - S^i - w_i S_i^i$$

is non decreasing in $w_i'$. Therefore, the expected profit is decreasing at all $w_i'$ such that $S^i(w_i', \widehat{w}_j) > r(S^j(\widehat{w}_j, w_i'))$. This is a sufficient condition for no profitable defection $w_i'$ from $\widehat{\mathbf{w}}$. ■

# F Proof of Proposition 6

Noting that $\Theta = \epsilon_s - \epsilon_d$, the proof holds by inspection upon substitution by rearranging terms in equations (20) and (22). $\square$

# G Proof of Proposition 7

The proof is constructed by considering price movements from the Bertrand equilibrium position under PTS and PTO. In the case of an interior solution, the Bertrand equilibrium for an agent facing a contract of the form in (23) is defined by the first-order necessary condition

$$p_i - w_i - t_i = \frac{S^i(\mathbf{w})}{S^i_i(\mathbf{w})} - \frac{D^i(\mathbf{p})}{D^i_i(\mathbf{p})}, \qquad i = 1, 2.$$

In the PTS game, evaluating the agent's input price condition (24) at the symmetric Bertrand equilibrium position $(w^B, p^B)$ gives

$$\Pi_i^{i,s}(w^B, p^B) \equiv \frac{S^i D^i_j S^i_i}{\Delta} \Theta \stackrel{s}{=} \Theta. \tag{33}$$

In the PTO game, evaluating the output price condition (25) at the symmetric Bertrand equilibrium position $(w^B, p^B)$ gives

$$\Pi_i^{i,o}(w^B, p^B) \equiv \frac{D^i D^i_i S^i_j}{\Sigma} \Theta \stackrel{s}{=} \Theta. \tag{34}$$

By Proposition 1, oligopoly profits rise with defections from the Bertrand equilibrium $(p', w')$ that involve $p' > p^B$ and $w' < w^B$. Letting the adjusted margin per unit of output of firm $i$ be denoted $\widehat{\Gamma}_i = \frac{p_i - w_i - t_i}{S^i}$, the equilibrium timing of the game is determined by the adjusted margins according to Proposition 4. $\square$